CSCanada
Studies in Mathematical Sciences
Vol. 3, No. 2, 2011, pp. 11-15
DOI: 10.3968/j.sms.1923845220110302.175

ISSN 1923-8444 [Print]
ISSN 1923-8452 [Online]
www.cscanada.net
www.cscanada.org

E-Cordial Labeling for Cartesian Product of Some Graphs

S. K. Vaidya[1,*]; N. B. Vyas[2]

[1]Department of Mathematics, Saurashtra University, Rajkot-360005, Gujarat, India
[2]Atmiya Institute of Technology & Science, Rajkot-360003, Gujarat, India
[*]Corresponding author.
Email: samirkvaidya@yahoo.co.in

Received 10 October, 2011; accepted 27 November, 2011

Abstract
We investigate E-cordial labeling for some cartesian product of graphs. We prove that the graphs $K_n \times P_2$ and $P_n \times P_2$ are E-cordial for n even while $W_n \times P_2$ and $K_{1,n} \times P_2$ are E-cordial for n odd.

Key words
E-Cordial labeling; Edge graceful labeling; Cartesian product

S. K. Vaidya, N. B. Vyas (2011). E-Cordial Labeling for Cartesian Product of Some Graphs. *Studies in Mathematical Sciences, 3*(2), 11-15. Available from: URL: http://www.cscanada.net/index.php/sms/article/view/j.sms.1923845220110302.175
DOI: http://dx.doi.org/10.3968/j.sms.1923845220110302.175

1. INTRODUCTION

We begin with finite, connected and undirected graph $G = (V(G),E(G))$ without loops and multiple edges. For standard terminology and notations we refer to West (2001). The brief summary of definitions and relevant results are given below.

Definition 1.1 If the vertices of the graph are assigned values subject to certain condition(s) then it is known as *graph labeling*.

Most of the graph labeling techniques trace their origin to graceful labeling introduced independently by Rosa (1967) and Golomb (1972) which is defined as follows.

Definition 1.2 A function f is called *graceful labeling* of graph G if $f : V(G) \to \{0, 1, 2, \ldots, q\}$ is injective and the induced function $f^*(e = uv) = |f(u) - f(v)|$ is bijective. A graph which admits graceful labeling is called a *graceful graph*.

The famous Ringel-Kotzig graceful tree conjecture and illustrious work by Kotzig (1973) brought a tide of labelig problems having graceful theme.

Definition 1.3 A graph G is said to be *edge-graceful* if there exists a bijection $f : E(G) \to \{1, 2, \ldots, |E|\}$ such that the induced mapping $f^* : V(G) \to \{0, 1, 2, \ldots, |V| - 1\}$ given by $f^*(x) = \sum f(xy)(mod|V|)$, $xy \in E(G)$.

Definition 1.4 A mapping $f : V(G) \to \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the *label* of vertex v of G under f.

Notations 1.5 For an edge $e = uv$, the induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Then

$$\left.\begin{array}{ll} v_f(i) = & number\ of\ vertices\ of\ G\ having\ label\ i\ under\ f \\ e_f(i) = & number\ of\ edges\ of\ G\ having\ label\ i\ under\ f^* \end{array}\right\} where\ i = 0\ or\ 1$$

Definition 1.6 A binary vertex labeling of graph G is called a *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called *cordial* if admits cordial labeling.

The concept of cordial labeling was introduced by Cahit (1987). He also investigated several results on this newly introduced concept.

Definition 1.7 A function $f : E(G) \rightarrow \{0, 1\}$ is called *E-cordial labeling* of graph G if the induced function $f^* : V(G) \rightarrow \{0, 1\}$ defined by $f^*(v) = \sum_{uv \in E(G)} f(uv)(mod\ 2)$ is such that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph is called *E-cordial* if admits E-cordial labeling.

Yilmaz and Cahit (1997) introduced E-cordial labeling as a weaker version of edge-graceful labeling and having bland of cordial labeling. They proved that the trees with n vertices, the complete graph K_n and cycle C_n are E-cordial if and only if $n \not\equiv 2(mod 4)$ while complete bipartite graph $K_{m,n}$ admits E-cordial labeling if and only if $m + n \not\equiv 2(mod 4)$

Devaraj (2004) has shown that $M(m, n)$ (the mirror graph of $K_{m,n}$) is E-cordial when $m + n$ is even while the generalized Petersen graph $P(n, k)$ is E-cordial when n is even. Vaidya and Vyas (2011) have proved that the mirror graphs of even cycle C_n, even path P_n and hypercube Q_k are E-cordial graphs.

Definition 1.8 Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The *cartesian product* of $G_1$and G_2 which is denoted by $G_1 \times G_2$ is the graph with vertex set $V = V_1 \times V_2$ consisting of vertices $V = \{u = (u_1, u_2), v = (v_1, v_2)/u$ and v are adjacent in $G_1 \times G_2$whenever $u_1 = v_1$ and u_2 adjacent to v_2 or u_1 adjacent to v_1 and $u_2 = v_2\}$

In this paper we have investigated some results on E-cordial labeling for cartesian product of some graphs.

2. MAIN RESULTS

Theorem-2.1: $K_n \times P_2$ is E-cordial for even n.

Proof: Let G be the graph $K_n \times P_2$ where n is even and $V(G) = \{v_{ij}/i = 1, 2, \ldots, n$ and $j = 1, 2\}$ be the vertices of graph G We note that $|V(G)| = 2n$ and $|E(G)| = n^2$ as $|V(K_n)| = n$ and $|E(K_n)| = \frac{n(n-1)}{2}$.

Define $f : E(G) \rightarrow \{0, 1\}$ as follows:

For $1 \leq i, k \leq n$

$$f(v_{i1}, v_{k1}) = 0;$$

$$f(v_{i2}, v_{k2}) = 1;$$

$$f(v_{i1}, v_{i2}) = \begin{cases} 1; & i \equiv 0\,(mod 2) \\ 0; & otherwise. \end{cases}$$

In view of the above defined labeling pattern f satisfies the conditions for E-cordial labeling as shown in *Table 1*. That is, $K_n \times P_2$ is E-cordial for even n.

Table 1

	vertex condition	*edge condition*
n	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = \frac{n^2}{2}$

Illustration 2.2: The E-cordial labeling for $K_4 \times P_2$ is shown in *Figure 1*.

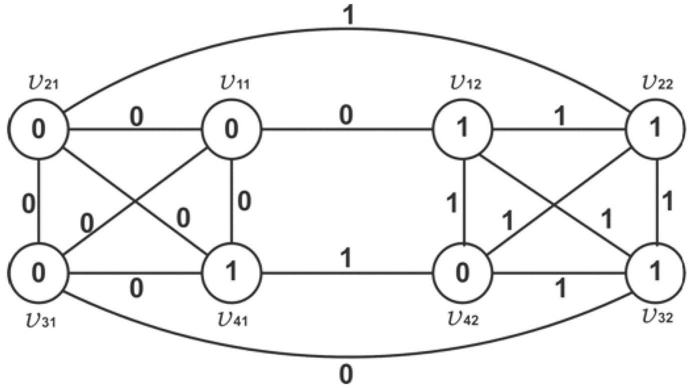

Figure 1

Theorem-2.3: $W_n \times P_2$ is E-cordial for odd n.
Proof: Let G be the graph $W_n \times P_2$ where n is odd and $V(G) = \{v_{ij}/i = 1, 2, \dots, n+1 \text{ and } j = 1, 2\}$ be the vertices of graph G. We note that $|V(G)| = 2(n+1)$ and $|E(G)| = 5n+1$as $|V(W_n)| = n+1$ and $|E(W_n)| = 2n$.

Def ne $f : E(G) \to \{0, 1\}$ as follows:

For $1 \leq i, k \leq n+1$

$$f(v_{i1}, v_{k1}) = 0; \; f(v_{i2}, v_{k2}) = 1;$$

$$\begin{aligned} f(v_{i1}, v_{i2}) &= 1; \quad i \equiv 0\,(mod 2) \\ &= 0; \quad otherwise. \end{aligned}$$

In view of the above def ned labeling pattern f satisf es conditions for E-cordial labeling as shown in *Table 2*. That is, $W_n \times P_2$ is E-cordial for odd n.

Table 2

	vertex condition	*edge condition*
n	$v_f(0) = v_f(1) = n+1$	$e_f(0) = e_f(1) = \frac{5n+1}{2}$

Illustration 2.4: The E-cordial labeling for $W_3 \times P_2$ is shown in *Figure 2*.

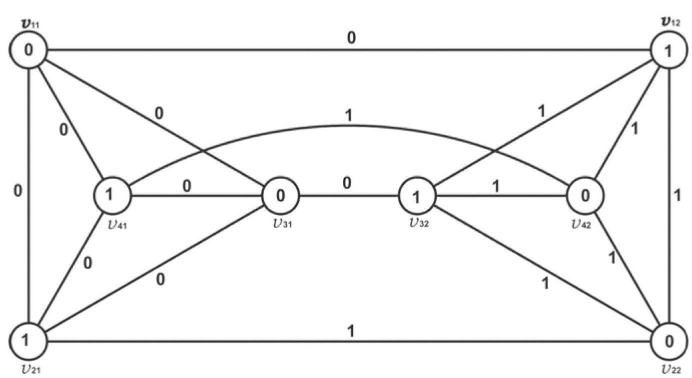

Figure 2

Theorem-2.5: $L_n = P_n \times P_2$(also known as ladder graph) is E-cordial for even n.
Proof: Let G be the graph $P_n \times P_2$ where n is even and $V(G) = \{v_{ij}/i = 1, 2, \dots, n \text{ and } j = 1, 2\}$ be the vertices of$G$. We note that $|V(G)| = 2n$ and $|E(G)| = 3n-2$. Def ne $f : E(G) \to \{0, 1\}$ as follows:

For $1 \leq i, k \leq n$

$$f(v_{i1}, v_{k1}) = 0;$$

$$f(v_{i2}, v_{k2}) = 1;$$

$$f(v_{i1}, v_{i2}) = \begin{cases} 1; & i \equiv 0 \,(mod 2) \\ 0; & otherwise. \end{cases}$$

In view of the above def ned labeling pattern f satisf es conditions for E-cordial labeling as shown in *Table 3*. That is, $P_n \times P_2$ is E-cordial for even n.

Table 3

	vertex condition	edge condition
n	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = \frac{3n-2}{2}$

Illustration 2.6: The E-cordial labeling for $P_4 \times P_2$ is shown in *Figure 3*.

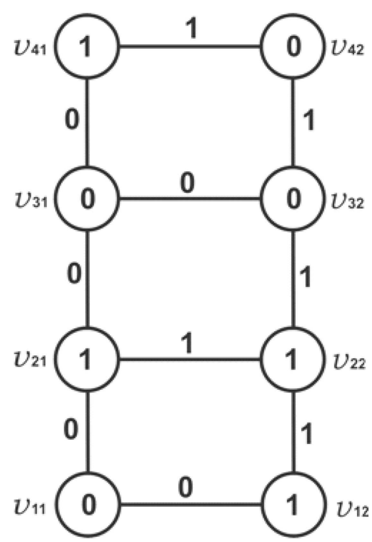

Figure 3

Theorem-2.7: $B_n = K_{1,n} \times P_2$(also known as book graph) is E-cordial for odd n.
Proof: Let G be the graph $K_{1,n} \times P_2$ where n is odd and $V(G) = \{v_{ij} / i = 1, 2, \ldots, n+1 \text{ and } j = 1, 2\}$ be the vertices of G. We note that $|V(G)| = 2(n+1)$ and $|E(G)| = 3n+1$. Def ne $f : E(G) \rightarrow \{0, 1\}$ as follows:

For $1 \leq i, k \leq n+1$

$$f(v_{i1}, v_{k1}) = 0;$$

$$f(v_{i2}, v_{k2}) = 1;$$

$$f(v_{i1}, v_{i2}) = \begin{cases} 1; & i \equiv 0 \,(mod 2) \\ 0; & otherwise. \end{cases}$$

In view of the above def ned labeling pattern f satisf es the conditions for E-cordial labeling as shown in *Table 4*. That is, $K_{1,n} \times P_2$is E-cordial for odd n.
Illustration 2.8: The E-cordial labeling for $K_{1,3} \times P_2$is shown in *Figure 4*.

Table 4

	vertex condition	*edge condition*
n	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = \frac{3n+1}{2}$

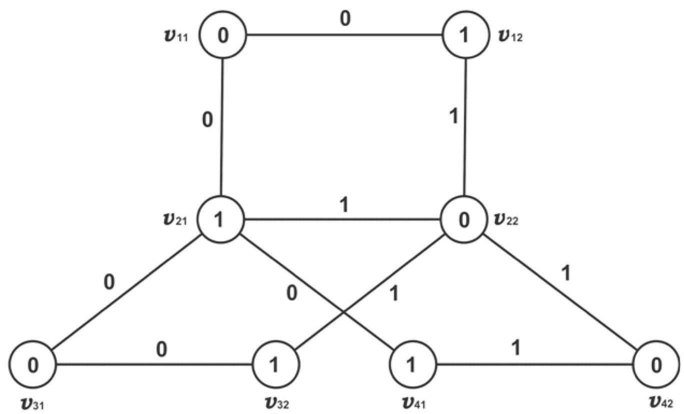

Figure 4

3. CONCLUDING REMARKS

Here we investigate E-cordial labeling for cartesian product of some graphs. Similar results can be derived for other graph families and in the context of different graph labeling problems is an open area of research.

REFERENCES

[1] Cahit I. (1987). Cordial Graphs: A Weaker Version of Graceful and Harmonious Graphs. *Ars Combinatoria, 23*, 201-207.

[2] Devraj J. (2004). On Edge-Cordial Graphs. *Graph Theory Notes of New York, XLVII*, 14-18.

[3] Gallian, J. A. (2010). A Dynamic Survey of Graph Labeling. *The Electronic Journal of Combinatorics, 17*(#DS6). Retrieved from: http://www.combinatorics.org/Surveys/ds6.pdf

[4] Golomb S. W. (1972). How to Number a Graph. In *Graph theory and Computing*, R. C. Read, ed., New York: Academic Press. pp.23-27.

[5] Kotzig A. (1973). On Certain Vertex Valuations of Finite Graphs. *Util. Math. 4*, 67-73.

[6] Rosa A. (1967). On Certain Valuations of the Vertices of a Graph. *Theory of graphs (Internat. Symposium, Rome, July 1966)*, Gordon and Breach, N. Y. and Dunod Paris, 349-355.

[7] Vaidya S. K. & Vyas, N. B. (2011). E-Cordial Labeling of Some Mirror Graphs, *International Journal of Contemporary Advanced Mathematics, 2*(1), 22-27. Retrieved from: http://www.ccsenet.org/journal/index.php/jmr/article/view/9863/8116

[8] Yilmaz R. & Cahit, I. (1997). E-Cordial Graphs, *Ars. Combinatoria, 46* , 251-266.

[9] West D. B. (2001), *Introduction to Graph Theory*, Prentice-Hall of India.