Annals of Pure and Applied Mathematics
Vol. 2, No. 1, 2012, 33-39
ISSN: 2279-087X (P), 2279-0888(online)
Published on 11 December 2012
www.researchmathsci.org

Annals of
Pure and Applied Mathematics

Antimagic Labeling in the Context of Switching of a Vertex

***S. K. Vaidya*[1] and *N. B. Vyas*[2]**

[1]Department of Mathematics, Saurashtra University,
Rajkot – 360005, Gujarat, INDIA. E-mail: samirkvaidya@yahoo.co.in

[2]Department of Mathematics, Atmiya Institute of Technology & Science,
Rajkot – 360005, Gujarat, INDIA. E-mail: niravbvyas@gmail.com

Received 30 November 2012; accepted 10 December 2012

Abstract. A graph with q edges is called antimagic if its edges can be labeled with 1, 2,…,q such that the sums of the labels of the edges incident to each vertex are distinct. Here we prove that the graphs obtained by switching of a pendant vertex in path P_n, switching of a vertex in cycle C_n, switching of a rim vertex in wheel W_n, switching of an apex vertex in helm H_n and switching of a vertex of degree 2 in fan f_n admit antimagic labeling.

AMS Mathematics Subject Classification (2010): **05C78, 05C38**

Keywords: Switching of a vertex, Antimagic labeling, Antimagic graphs.

1. Introduction

We begin with a finite, connected and undirected graph $G = (V(G),E(G))$ without loops and multiple edges. Throughout this paper $|V(G)|$ and $|E(G)|$ respectively denote the number of vertices and number of edges in G. For any undefined notation and terminology we rely upon Gross and Yellen[3]. A brief summary of definitions and existing results is provided in order to maintain the compactness.

Definition 1.1. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges) then the labeling is called a *vertex labeling* (or an *edge labeling*).

According to Beineke and Hegde[1] labeling of discrete structure is a frontier between graph theory and theory of numbers. For an extensive survey of graph labeling as well as bibliographic references there in we refer to Gallian[2].

Definition 1.2. A graph with q edges is called *antimagic* if its edges can be labeled with 1, 2,…,q such that the sums of the labels of the edges incident to each vertex are distinct.

The concept of antimagic graph was introduced by Hartsfield and Ringel[4]. They showed that paths P_n (n ≥ 3), cycles, wheels, and complete graphs K_n (n ≥ 3) are antimagic. They conjectured that

(i) *all trees except K_2 are antimagic,*
(ii) *all connected graphs except K_2 are antimagic*.

These conjectures are unsettled till today.

Definition 1.3. A *vertex switching* G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges to v and adding edges joining v to every other vertex which are not adjacent to v in G.

Definition 1.4. The *wheel graph* W_n is defined to be the join K_1+C_n. The vertex corresponding to K_1 is known as *apex vertex* and vertices corresponding to cycle are known as *rim vertices* while the edges corresponding to C_n are known as *rim edges*. We continue to recognize apex of wheel as the apex of the respective graphs obtained from wheel.

Definition 1.5. The *helm* H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.

Definition 1.6. A *fan* graph f_n is obtained by P_n+K_1.

In the following section we will investigate some new results on Antimagic labeling of graphs.

2. Main Results

Theorem 2.1. Switching of a pendant vertex in a path P_n is antimagic.
Proof: Let $v_1,v_2,\ldots,v_n$ be the vertices of P_n and G_v denotes the graph obtained by switching of a pendant vertex v of $G = P_n$. Without loss of generality let the switched vertex be v_1. We note that $|V(G_{v_1})| = n$ and $|E(G_{v_1})| = 2n-4$. We define $f : E\left(G_{v_1}\right) \to \{1,2,...,2n-4\}$ as follows:

For $2 \le i \le n-1$:
$f(v_i v_{i+1}) = i-1$;
For $3 \le i \le n$:
$f(v_1 v_i) = n+i-4$,
Above defined edge labeling function will generate all distinct vertex labels as per the definition of antimagic labeling. Hence the graph obtained by switching of a pendant vertex in a path P_n is antimagic.

Illustration 2.2. In *Figure* 1 the graph obtained by switching of a vertex v_1 in path P_5 and its antimagic labeling is shown.

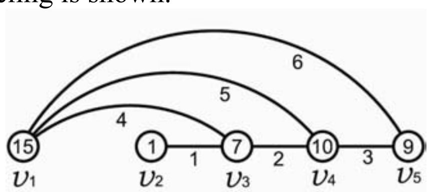

Figure 1

Theorem 2.3. Switching of a vertex in cycle C_n is antimagic.

Proof. Let $v_1,v_2, \ldots,v_n$ be the successive vertices of C_n and G_v denotes graph obtained by switching of vertex v of $G= C_n$. Without loss of generality let the switched vertex be v_1. We note that $\left|V\left(G_{v_1}\right)\right| = n$ and $\left|E\left(G_{v_1}\right)\right| = 2n-5$. We define $f : E\left(G_{v_1}\right) \rightarrow \{1,2,...,2n-5\}$ as follows:

For $3 \le i \le n$:

$f(v_1v_i) = 2(i-2)$;

$f(v_{i-1}v_i) = 2i-5$,

Above defined edge labeling function will generate all distinct vertex labels as per the definition of an antimagic labeling. Hence the graph obtained by switching of a vertex in a cycle C_n is antimagic.

Illustration 2.4. In *Figure* 2 the graph obtained by switching of a vertex v_1 in cycle C_7 and its antimagic labeling is shown.

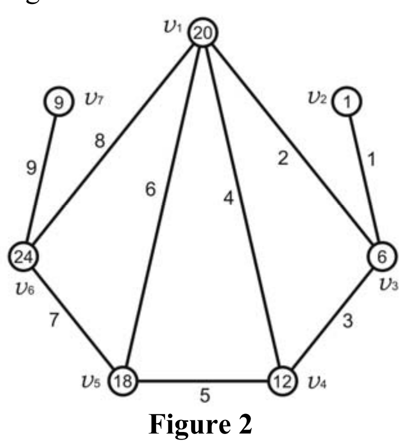

Figure 2

Theorem 2.5. Switching of a rim vertex in a wheel W_n is antimagic.

Proof. Let v as the apex vertex and $v_1,v_2, \ldots ,v_n$ be the rim vertices of wheel W_n. Let G_{v_1} denotes graph obtained by switching of a rim vertex v_1 of $G = W_n$. We note that

$\left|V\left(G_{v_1}\right)\right| = n+1$ and $\left|E\left(G_{v_1}\right)\right| = 3n-6$. We define $f : E\left(G_{v_1}\right) \to \{1, 2, ..., 3n-6\}$ as follows.

For $2 \le i \le n-1$:

$f(v_i v_{i+1}) = i-1$;

For $2 \le i \le n$:

$f(vv_i) = 2n-i-1$;

For $3 \le i \le n-1$:

$f(v_1 v_i) = 2n+i-5$,

Above defined edge labeling function will generate all distinct vertex labels as per the definition of an antimagic labeling. Hence the graph obtained by switching of a rim vertex in a wheel W_n is antimagic.

Illustration 2.6. In *Figure* 3 the graph obtained by switching of a vertex v_1 in wheel W_8 and its antimagic labeling is shown.

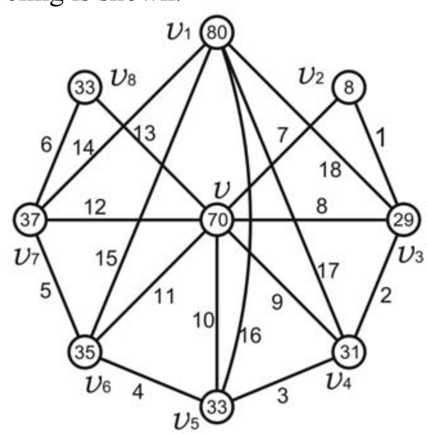

Figure 3

Theorem 2.7. Switching of an apex vertex in helm H_n is antimagic.

Proof. Let H_n be a helm with v as the apex vertex, $v_1, v_2, ..., v_n$ be the vertices of cycle and $u_1, u_2, ..., u_n$ be the pendant vertices. Let G_v denotes graph obtained by switching of an apex vertex v of $G=H_n$. We note that $\left|V\left(G_v\right)\right| = 2n+1$ and $\left|E\left(G_v\right)\right| = 3n$. We define $f : E\left(G_v\right) \to \{1, 2, ..., 3n\}$ as follows.

Case 1: $n \equiv 0 \pmod 3$; $n \ne 3$

$f(vu_1) = 2$;

$f(v_1 u_1) = 1$;

$f(v_1 v_2) = 3$;

For $2 \le i \le n$:

$f(vu_i) = 3i - 2$;

$f(v_i u_i) = 3i - 1$;

$f(v_i v_{i+1}) = 3i$ where $(v_{n+1} = v_1)$

The case when $n = 3$ is to be dealt separately and the graph is labeled as shown in *Figure* 4.

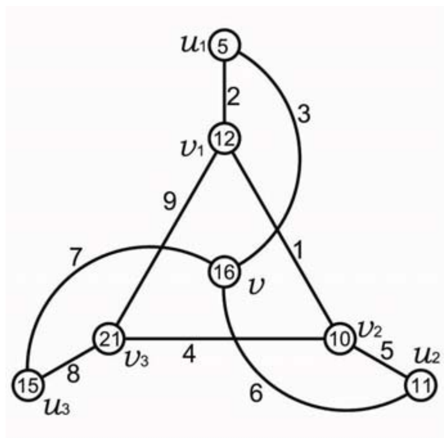

Figure 4

Case 2: $n \equiv 1, 2 \pmod 3$

For $1 \le i \le n$:

$f(vu_i) = 3i - 2$;

$f(v_i u_i) = 3i - 1$;

$f(v_i v_{i+1}) = 3i$ where $(v_{n+1} = v_1)$

Above defined edge labeling function will generate all distinct vertex labels as per the definition of an antimagic labeling. Hence the graph obtained by switching of an apex vertex in a helm H_n is antimagic.

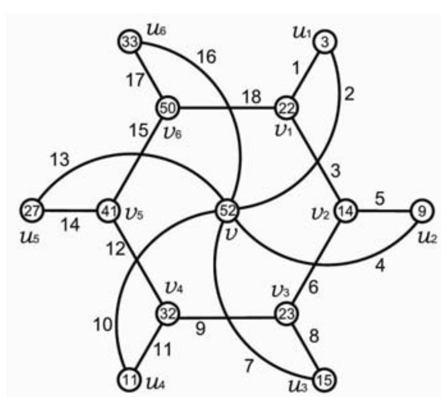

Figure 5

Illustration 2.8. In *Figure* 5 the graph obtained by switching of an apex vertex v in helm H_5 and antimagic labeling is shown.

Theorem 2.9. Switching of a vertex having degree 2 in fan f_n is antimagic.

Proof. Let v as the apex vertex and $v_1, v_2, \ldots, v_n$ be the vertices of fan f_n. Let G_{v_1} denotes graph obtained by switching of a vertex v_1 having degree 2 of $G = f_n$. We note that $\left|V\left(G_{v_1}\right)\right| = n+1$ and $\left|E\left(G_{v_1}\right)\right| = 3n-5$.

We define $f : E\left(G_{v_1}\right) \to \{1,2,...,3n-5\}$ as follows.

For $2 \le i \le n$:

$f(vv_i) = i-1$;

For $2 \le i \le n-1$:

$f(v_iv_{i+1}) = n+i-2$;

For $3 \le i \le n$:

$f(v_1v_i) = 2n+i-5$;

Above defined edge labeling function will generate all distinct vertex labels as per the definition of an antimagic labeling. Hence the graph obtained by switching of a vertex having degree 2 in fan f_n is antimagic.

Illustration 2.10. In *Figure* 6 the graph obtained by switching of a vertex v_1 having degree 2 in fan f_5 and its antimagic labeling is shown.

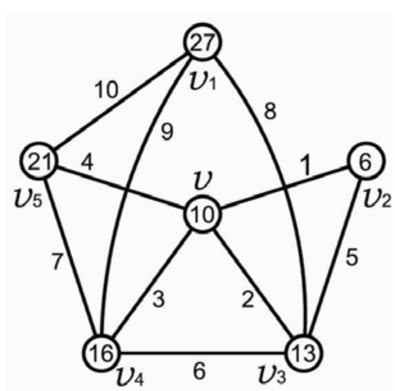

Figure 6

3. Concluding Remarks

The investigations reported here is an effort to relate graph operations and antimagic labeling. To investigate some characterization(s) or sufficient condition(s) for any graph to be antimagic is an open area of research.

REFERENCES

1. L. W. Beineke and S. M. Hegde, Strongly Multiplicative graphs, *Discuss. Math. Graph Theory*, vol. 21, pp. 63-75, 2001.
2. J. A. Gallian, A Dynamic Survey of Graph Labeling, *The Electronics Journal of Combinatorics*, vol. 18 (#DS6), 2011.
   Available: http://www.combinatorics.org/Surveys/ds6.pdf
3. J. Gross and J. Yellen, Graph Theory and its Applications, *CRC Press*, 1999.
4. N. Hartsfield and G. Ringel, Pearls in Graph Theory, *Academic Press*, Boston, 1990.