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EVEN MEAN LABELING FOR PATH AND BISTAR RELATED GRAPHS

S. K. Vaidya [a,*] **and N. B. Vyas** [b]

[a] Department of Mathematics, Saurashtra University, Rajkot - 360005, Gujarat, India
[b] Atmiya Institute of Technology and Science, Rajkot - 360005, Gujarat, India

∗ **Corresponding author: S. K. Vaidya** Tel.: +91-9825292539; e-mail: samirkvaidya@yahoo.co.in
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ABSTRACT

An even mean labeling is a variant of mean labeling. Here we investigate even mean labeling for path and bistar related graphs.

Keywords: *Shadow graph; Splitting graph; Middle graph; Even Mean labeling.*
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1. INTRODUCTION

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (an *edge labeling*).

According to Beineke and Hegde[1] labeling of discrete structure is a frontier between graph theory and theory of numbers. A latest survey on various graph labeling problems can be found in Gallian[2].

The function f is called *mean labeling* of graph G if $f : V(G) \rightarrow \{0, 1, 2, \ldots, |E(G)|\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1, 2, \ldots, |E(G)|\}$ defined as

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even;} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

is bijective. A graph which admits mean labeling is called a *mean graph*.

The mean labeling was introduced by Somasundaram and Ponraj [3] and they proved that the graphs P_n, C_n, $P_n \times P_m$, $P_m \times C_n$ are mean graphs. Vaidya and Lekha [4] proved that the graphs P_m $[P_2]$, P_n^2, $M(P_n)$ and some cycle related graphs admit mean labeling.

According to Pricilla [5], function f is called *even mean labeling* of graph G if $f : V(G) \rightarrow \{2,4, \ldots, 2|E(G)|\}$ is injective and each edge uv assigned the label $\frac{f(u)+f(v)}{2}$ such that the resulting edge labels are distinct.

For a connected graph G, let G' be the copy of G then shadow graph $D_2(G)$ is obtained by joining each vertex u in G to the neighbours of the corresponding vertex u' in G'. The *middle graph* $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it. The *total graph* $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G. For a graph G the *splitting graph* $S'(G)$ is obtained by adding new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$ where $N(v)$ and $N(v')$ are the neighbourhood sets of v and v' respectively. Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k) \cap N(v''_k) = v_k$. A *vertex switching* G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges to v and adding edges joining v to every other vertex which are not adjacent to v in G. *Square of* a graph G denoted by G^2 has the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance at most 2 apart in G. *Cube of* a graph G denoted by G^3 has the same vertex set as of G and two vertices are adjacent in G^3 if they are at a distance at most 3 apart in G. The *double fan* DF_n is obtained by P_n+2K_1. For any undefined term in graph theory we rely upon West [6].

2. RESULTS

Theorem 2.1. The graph $D_2(P_n)$ is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n and $v'_1, v'_2, \ldots, v'_n$ be the newly added vertices corresponding to the vertices $v_1, v_2, \ldots, v_n$ in order to obtain $D_2(P_n)$. Denoting $G = D_2(P_n)$ then $|V(G)| = 2n$ and $|E(G)| = 4(n-1)$.

We define $f : V(G) \rightarrow \{2,4,\ldots,2|E(G)|\}$ as follows.

For $1 \le i \le n$:

$$f(v_i) = \begin{cases} 8i-6, & i \equiv 1 (\bmod 2); \\ 8i-12, & \text{otherwise.} \end{cases}$$

$$f(v'_i) = \begin{cases} 8i, & i \equiv 1 (\bmod 2); \\ 8i-10, & \text{otherwise.} \end{cases}$$

The above defined function f provides an even mean labelling for $D_2(P_n)$. That is, $D_2(P_n)$ is an even mean graph. □

Illustration 2.2. Shadow graph of path P_5 and its even mean labeling is shown in Fig.1.

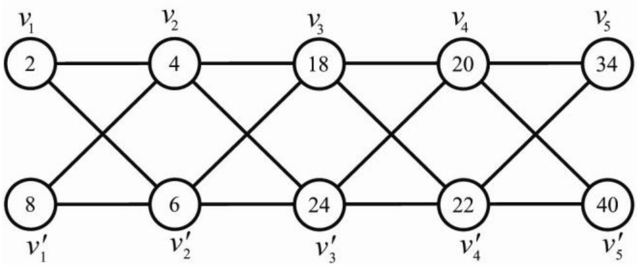

Fig.1

Theorem 2.3. Middle graph of path P_n is an even mean graph.

Proof. Let $v_1, v_2, ..., v_n$ be the vertices and $e_1, e_2, ..., e_{n-1}$ be the edges of path P_n and $G = M(P_n)$ be the middle graph of path P_n. According to the definition of middle graph $V(M(P_n)) = V(P_n) \cup E(P_n)$ and $E(M(P_n))$ = $\{v_i e_i; 1 \leq i \leq n-1, v_i e_{i-1}; 2 \leq i \leq n, e_i e_{i+1}; 1 \leq i \leq n-2\}$. Here $|V(G)| = 2n-1$ and $|E(G)| = 3n-4$.

We define $f : V(G) \rightarrow \{2, 4, \ldots, 2|E(G)|\}$ as follows.

For $1 \leq i \leq n$:

$f(v_i) = 4i - 2$;

For $1 \leq i \leq n-1$:

$f(e_i) = 4i$;

The above defined function *f provides* an even mean labeling for $M(P_n)$. Hence, $M(P_n)$ is an even mean graph. □

Illustration 2.4. $M(P_5)$ and it's even mean labeling is shown in Fig.2.

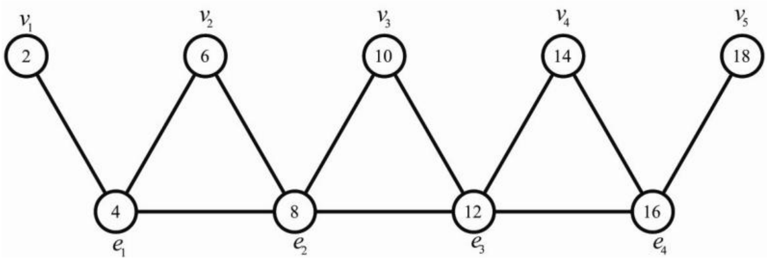

Fig.2

Theorem 2.5. Total graph of path P_n is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n and $G = T(P_n)$ be the total graph of path P_n with $V(T(P_n))$ = $V(P_n) \cup E(P_n)$ and $E(T(P_n))$ = $\{v_i v_{i+1}; 1 \leq i \leq n-1, v_i e_i; 1 \leq i \leq n-1,$ $e_i e_{i+1}; 1 \leq i \leq n-2, v_i e_{i-1}; 2 \leq i \leq n\}$. Here $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$.

We define $f : V(G) \rightarrow \{2, 4, \ldots, 2|E(G)|\}$ as follows.

For $1 \leq i \leq n$:

$f(v_i) = 4i - 2$;

For $1 \leq i \leq n-1$:

$f(e_i) = 4i$;

The above defined function f provides an even mean labeling for $T(P_n)$. Hence, $T(P_n)$ is an even mean graph. □

Illustration 2.6. $T(P_6)$ and its even mean labeling is shown in Fig.3.

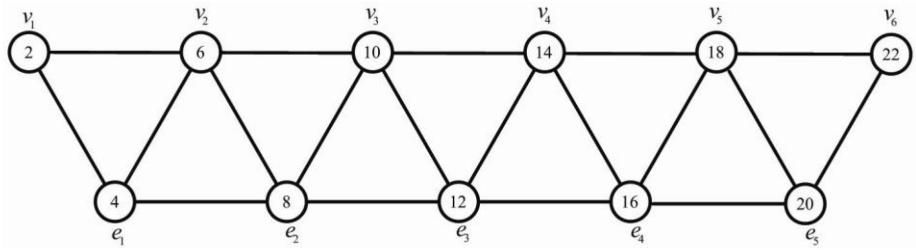

Fig.3

Theorem 2.7. Splitting graph of path P_n is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n. Let $v_1', v_2', \ldots, v_n'$ be the newly added vertices to form the splitting graph of path P_n. Let $G = S'(P_n)$ be the splitting graph of path P_n. $V(S'(P_n)) = \{v_i\} \cup \{v_i'\}, 1 \le i \le n$ and $E(S'(P_n))$ = $\{v_i' v_{i+1}; 1 \le i \le n-1, v_i' v_{i-1}; 2 \le i \le n, v_i v_{i+1}$ $; 1 \le i \le n-1\}$. Here $|V(G)| = 2n$ and $|E(G))| = 3n-3$.

We define $f : V(G) \to \{2, 4, \ldots, 2|E(G)|\}$ as follows.

For $1 \le i \le n$:

$$f(v_i) = \begin{cases} 4i, & i \equiv 1(\bmod 2); \\ 4i-2, & \text{otherwise.} \end{cases}$$

$$f(v_i') = \begin{cases} 4i-2, & i \equiv 1(\bmod 2); \\ 4i, & \text{otherwise.} \end{cases}$$

The above defined function f provides an even mean labeling for $S'(P_n)$. Hence, $S'(P_n)$ is an even mean graph. □

Illustration 2.8. $S'(P_6)$ and its even mean labeling is shown in Fig.4.

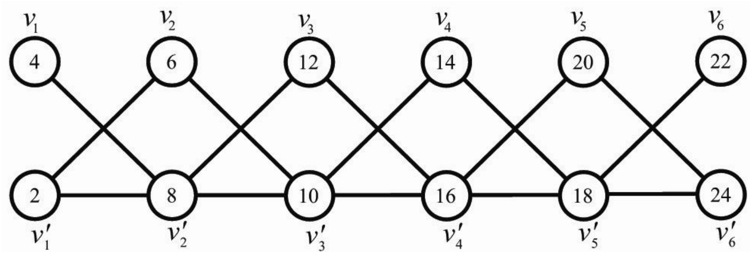

Fig.4

Theorem 2.9. Duplicating each vertex by an edge in path P_n is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n. Let G be the graph obtained by duplicating each vertex v_i of P_n by an edge $v_i' v_i''$ at a time , where $1 \le i \le n$. Note that $|V(G)| = 3n$ and $|E(G)| = 4n-1$.

We define $f : V(G) \to \{2, 4, \ldots, 2|E(G)|\}$ as follows.

For $1 \le i \le n$:

$f(v_i) = 8i - 6;$

$f(v_i') = 8i - 4;$

$f(v_i'') = 8i - 2;$

The above defined function f provides an even mean labeling for graph G. Hence, duplicating each vertex by edge in path P_n is an even mean graph. □

Illustration 2.10. Duplicating each vertex by edge in path P_7 and its even mean labeling is shown in Fig.5.

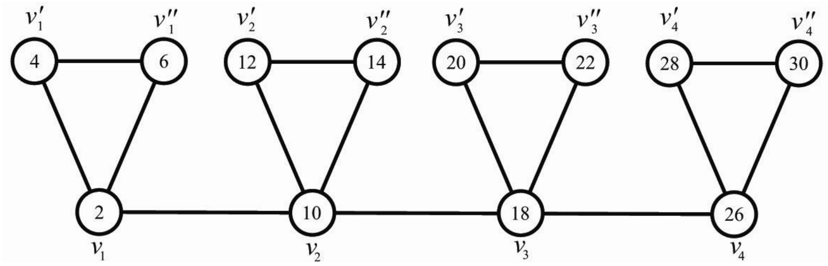

Fig.5

Theorem 2.11. Switching of a pendant vertex in path P_n is an even mean graph.

Proof. Let $v_1, v_2, ..., v_n$ be the vertices of P_n and G_v denotes the graph obtained by switching of a pendant vertex v of $G = P_n$. Without loss of generality let the switched vertex be v_1. We note that $|V(G_{v_1})| = n$ and $|E(G_{v_1})| = 2n - 4$.

We define $f : V(G_{v_1}) \rightarrow \{2, 4, ..., 4n - 8\}$ as follows:

$f(v_1) = 2$;

For $2 \le i \le n$:

$f(v_i) = 4n - 4 - 2i$;

The above defined function f provides an even mean labeling for G_{v_1}. Hence, the graph obtained by switching of a pendant vertex in a path P_n is an even mean graph. □

Illustration 2.12. Switching of a pendant vertex in path P_5 and its even mean labeling is shown in Fig.6.

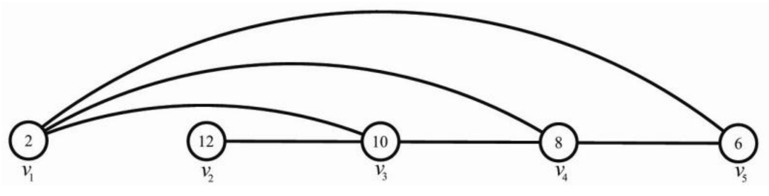

Fig.6

Theorem 2.13. P_n^2 is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n. Let $G = P_n^2$ then note that $|V(G)| = n$ and $|E(G)| = 2n - 3$.

We define $f : V(G) \rightarrow \{2, 4, \ldots, 4n - 6\}$ as follows.

For $1 \le i \le n$:

$f(v_i) = 2i$;

The above defined function f provides an even mean labeling for P_n^2. Hence, P_n^2 is an even mean graph. □

Illustration 2.14. P_6^2 and its even mean labeling is shown in Fig.7.

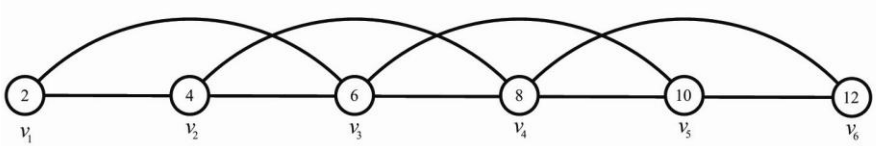

Fig.7

Theorem 2.15. P_n^3 is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of path P_n. Let $G = P_n^3$ then note that $|V(G)| = n$ and $|E(G)| = 3n - 6$.

We define $f : V(G) \to \{2, 4, \ldots, 6n - 12\}$ as follows.

For $1 \le i \le n$:

$$f(v_i) = \begin{cases} 12\left\lceil \frac{i}{4} \right\rceil - 10, & i \equiv 1 (\bmod 4); \\ 12\left\lceil \frac{i}{4} \right\rceil - 8, & i \equiv 2 (\bmod 4); \\ 12\left\lceil \frac{i}{4} \right\rceil - 6, & i \equiv 3 (\bmod 4); \\ 12\left\lceil \frac{i}{4} \right\rceil - 2, & i \equiv 0 (\bmod 4). \end{cases}$$

The above defined function f provides an even mean labeling for P_n^3. Hence, P_n^3 is an even mean graph. □

Illustration 2.16. P_5^3 and it's even mean labeling is shown in Fig.8.

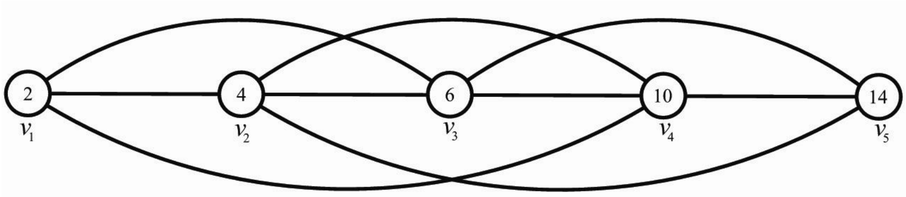

Fig.8

Theorem 2.17. $B_{n,n}^2$ is a mean graph.

Proof. Consider $B_{n,n}$ with vertex set $\{u, v, u_i, v_i, 1 \le i \le n\}$ where u_i, v_i are pendant vertices. Let G be the graph $B_{n,n}^2$. Then $|V(G)| = 2n + 2$ and $|E(G)| = 4n + 1$.

We define $f : V(G) \to \{2, 4, \ldots, 8n + 2\}$ as follows.

$f(u) = 8n + 2$;

$f(v)=2$;

For $1\leq i\leq n$:

$f(v_i)=2+2i$;

$f(u_i)=8n+2-2i$;

The above defined function *f* provides an even mean labeling for $B^2_{n,n}$. Hence, $B^2_{n,n}$ is an even mean graph. □

Illustration 2.18. $B^2_{5,5}$ and its even mean labeling is shown in Fig.9.

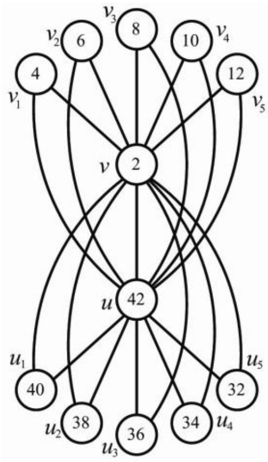

Fig.9

Theorem 2.19. $S'(B_{n,n})$ is an even mean graph.

Proof. Consider $B_{n,n}$ with the vertex set $\{u,v,u_i,v_i,1\leq i\leq n\}$ where u_i,v_i are pendant vertices. In order to obtain $S'(B_{n,n})$, add u',v',u_i',v_i' vertices corresponding to *u*, *v*, u_i, v_i where, $1\leq i\leq n$. If $G=S'(B_{n,n})$ then $|V(G)|=4(n+1)$ and $|E(G)|=6n+3$.

We define $f:V(G)\rightarrow\{2,4,\ldots,12n+6\}$ as follows.

$f(u)=6n+2$; $f(v)=12n+6$;

$f(u')=2$; $f(v')=6n+4$;

For $1\leq i\leq n$:

$f(u_i)=2i+2$;

$f(u_i')=2n+2+2i$;

$f(v_i)=6n+4+2i$;

$f(v_i')=8n+4+2i$;

The above defined function *f* provides an even mean labeling for $S'(B_{n,n})$. Hence, $S'(B_{n,n})$ is an even mean graph. □

Illustration 2.20. $S'(B_{4,4})$ and it's even mean labeling is shown in Fig.10.

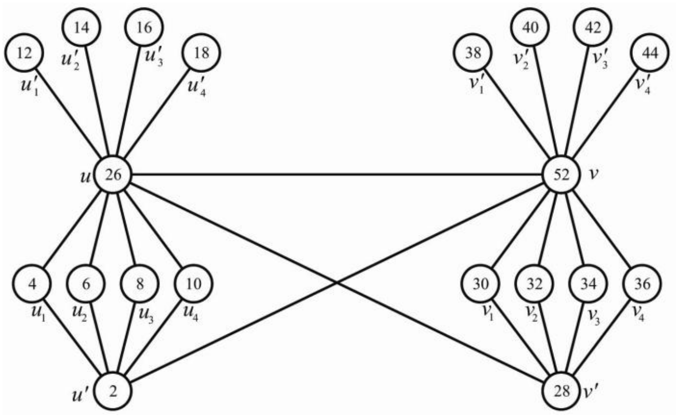

Fig.10

Theorem 2.21. DF_n is an even mean graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n for n even. Vertices u and v are added to obtain $DF_n = P_n + 2K_1$. We note that $|V(DF_n)| = n+2$ and $|E(DF_n)| = 3n-1$.

We define $f : V(DF_n) \to \{2, 4, \ldots, 6n-2\}$ as follows.

$f(u) = 2$;

$f(v) = 6n-2$;

For $1 \le i \le n$:

$f(v_i) = 2n + 2i - 2$;

The above defined function *f* provides an even mean labeling for DF_n. Hence, DF_n is an even mean graph. □

Illustration 2.22. DF_6 and it's even mean labeling is shown in Fig.11.

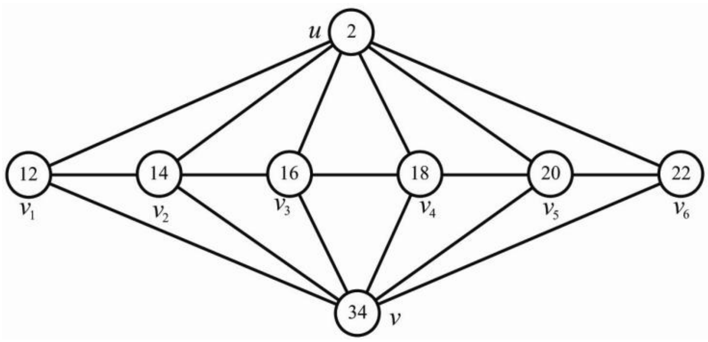

Fig.11

CONCLUSIONS

It is always interesting to find out graph or graph families which admit a particular labeling. Here we investigate some new graph families which admit even mean labeling. To investigate similar results for other graph families is an open area of research.

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