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# Development of a Simplified Mathematical Modelling for the Job Shop Scheduling to Calculate Makespan

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## *Abstract*
*In this paper, mathematical modelling is prepared for job shop scheduling problem. Makespan is calculated using maximum machine time. Matlab algorithm is developed for mathematical modelling. Formula is tested using various case studies. Gantt chart is prepared for graphical representation of solution. Results have been verified for mathematical modelling using Gantt chart approach.*

***Keywords:*** *Job-Shop Scheduling, Mathematical modeling, Gantt chart, Makspan*

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## INTRODUCTION
Job shop scheduling is essential tool for improvement of productivity of any industry. The Job Shop Scheduling Problem is one of the NP hard optimization problems, and it is difficult to obtain the exact optimal solution. Dispatching rules are widely accepted in the industry because of the ease of implementation, satisfactory performance, Low computation requirement, and flexibility to incorporate domain knowledge and expertise.

In recent years, optimization algorithms such as simulated annealing (SA), genetic algorithm (GA), tabu search (TS), ant colony optimization (ACO) particle swarm optimization (PSO) and artificial bee colony (ABC) have played a significant role in solving small-scale job shop scheduling problems [1].

### Literature Review
The Job Shop Scheduling Problem (JSSP) is one of the most popular scheduling models existing in practice which is among the hardest combinatorial optimization problems. Guo et al. proposed a universal mathematical model of the JSSP for apparel assembly process [2]. The objective of this model was to minimize the total penalties of earliness and tardiness by deciding when to start each order's production and how to assign the operations to machines (operators).

Genetic optimization process was used to solve the model. Seda proposed a mathematical model for job shop scheduling problems based on mixed integer programming formulations [3]. Moghaddas and Houshmand proposed a mathematical model for job shop scheduling with sequence dependent setup times [4]. Mahavi et al. developed a model for a single machine bi-criterion scheduling problem with deteriorating jobs with the aim of minimizing total tardiness and work in process costs [5]. Shapiro has presented mathematical programming models and solution methods that have been applied to several types of production planning and scheduling problems [6].

Pan has provided a review and comparison of mixed-integer linear programming (MILP) formulations for job-shop, Blazewichz et al. is the first one that focuses on mathematical models for scheduling problems [7,8]. They presented mathematical programming formulations for single-machine, parallel-machine and job shop scheduling problems. Bowman and Dantzig not only still retain their original forms but have also been pushed into neglect [9,10]. This may be due to their low efficiency and their failure to solve even small sized problems.

**Research Gap**

Various authors in these review has presented mathematical modeling regarding job shop scheduling but due to complexity of mathematics, it is difficult to convert it into algorithm for development of Matlab code so there is scope to simplify the mathematical modeling and Development of algorithm to calculate Makespan formula for job shop scheduling which can be further utilized for optimization purpose.

## MATHEMATICAL MODELING

Mathematical modeling can be generated by using following steps.

1. Design parameters
2. Design variable
3. Objective function
4. Constraints and Assumptions

**Design Parameters**

Design parameters indicates the total number of variable required to describe the problem

List of design variable are as follows:

1. n = no. of job
2. m = no. of machines
3. Xk = kth job up to i job or matrix which indicates the selection sequence.
4. T= time matrix for each job with all operation
5. MN = matrix for machine numbers for all operation of each job
6. Z = maximum completion time from each machine
7. $MTI_k$ = machine initial time for $k^{th}$ machine before each operation
8. $MTF_k$ = machine final time for $k^{th}$ machine after completion of each operation
9. $JTI_k$ = job initial time for job k before each operation
10. $JTF_k$ = job final time for job k after completion of each operation
11. $OC_k$ = operation count for each job k
12. $F_x$ = operation number for each Xk
13. $E_x$ = job number for each Xk

**Design Variables**

Design variables are selected to decide the job operation processing sequence. This can be evaluated using objective function.

$X_k$ = job-operation sequence number

$X_k = X_1\ X_2\ X_3\ X_4\ X_5\ X_6\ X_7\ X_8\ X_9 \ldots\ldots X_k$

1-1 1-2 1-3 2-1 2-2 2-3 3-1 3-2 3-3

Or 1 2 3 4 5 6 7 8 9

Here, $X_1$=1 =>**1-1** is **job-1 operation-1**

$X_2$=4 =>**2-1** is **job-2 operation-1**

All values should be different and integer

All, $X_k$>0 and k = m x n.

**Objective Function**

Makspan can be calculated using mathematical equation which is represented by objective function. Here objective function has been developed based on time required to complete all the jobs for different machines which also can be calculated based on job completion time. So after completion of all the operation on each machine, maximum of machine time will be considered as a makspan.

Maximum Completion time,

$Z = \text{maximum } \{MTF_1, MTF_2, MTF_3 \ldots MTF_m\}$

Where,

$MTF_k$ = machine final time for kth machine after completion of all the operation.

M = number of machines.

MTF and MTI for each machine will be evaluated for each operation and final MTF will be considered for the objective.

$MTF = MTI + T_x$

$T_x$ = processing time of X (which identifies job E and operation F)

**Constraints and Assumption**

Some industrial and process restrictions are considered to achieve the feasibilities of method.

1. $MTI_k$ (i+1)= maximum $\{MTF_k(i), JTF_k(i)\}$

Where, I is the operation number.

2. Operation sequence must be followed ($F_x$= $OC_k$ for each operation).
3. Initial value of machine final time and job final time before first operation is zero ($MTF_k$= 0 and $JTF_k$ = 0).

  

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## CASE PROBLEM ON JOB SHOP SCHEDULING

The following case study of 3 Jobs 3 Machines has been considered to evaluate the mathematical modeling. And results obtained are as follows.

Number of job = 3
Number of machines = 3
Number of operation = 3

### Machine Time Data:

| | Operation 1 (min) | Operation 2 (min) | Operation 3 (min) |
|---|---|---|---|
| Job 1 | 5 | 10 | 4 |
| Job 2 | 4 | 5 | 6 |
| Job 3 | 5 | 3 | 7 |

### Machine Number Data:

| | Operation 1 | Operation 2 | Operation 3 |
|---|---|---|---|
| Job 1 | M1 | M2 | M3 |
| Job 2 | M3 | M1 | M2 |
| Job 3 | M3 | M2 | M1 |

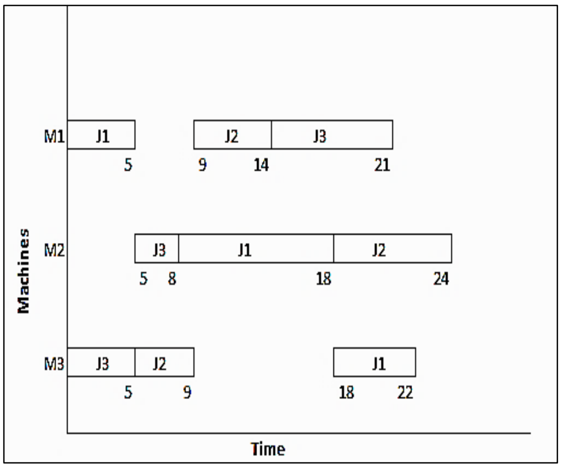
For the job sequence X = 7 4 1 5 8 9 2 6 3
Or 31 21 11 22 32 33 23 13 and the solution is = 24 min

| Jobs | Processing time (min) * | | | Due dates (min) |
|---|---|---|---|---|
| 1 | 1 (005) | 2 (010) | 3 (004) | 18 |
| 2 | 3 (004) | 1 (005) | 2 (006) | 24 |
| 3 | 3 (005) | 2 (003) | 1 (007) | 16 |

*Makspan = 24 min.*

### Gantt Chart for Case Problem on Job Shop Scheduling

Gantt chart has been prepared for similar problem of 3 Jobs 3 Machines to verify the results of methodology.




### Gant Chart for 3 Jobs 3 Machines

| Job | Due Date | Completion Time | Tardiness |
|---|---|---|---|
| J-1 | 18 | 22 | 4 |
| J-2 | 24 | 24 | - |
| J-3 | 16 | 21 | 5 |

- Maximum Tardiness = 5
- Makspan = 24 min.

## CONCLUSIONS

Mathematical model has been developed using operation engineering terminology for the job shop scheduling problem to estimate makspan. The mathematical formula has been successfully validated using Gantt chart approach. Developed method can be used for optimization of makspan.

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