Perfect Domination Number of a Cycle graph C_n and its Corona product with another Cycle graph C_{n-1}

Dr. Tushharkumar Bhatt [1] Dr. Hiren Kavathiya [2] Mr. Milan Gohel [3]

1. Assistant professor, Department of Science and Humanities, Atmiya University, Rajkot
2. Assistant Professor, Department of Information and Technology, Atmiya University, Rajkot
3. Assistant professor, Department of Information and Technology, Atmiya University, Rajkot.

Abstract - According to the research paper on Perfect Dominating Sets by Marilynn Livingston and Quentin F. stout[1]they have been constructed the PDSs in families of graphs arising from the interconnected networks of parallel computers also contained perfect domination numbers of trees, dags, series-parallel graphs, meshes, tori, hypercubes, cube connected cycles and de Bruijin graphs and give linear algorithms for determining if a PDS exist, and generate a PDS when one does. They also proved that 2 and 3-dimensional hypercube graph having infinitely many PDSs. In this paper we are trying to apply their concept on cycle graphs and obtained their perfect domination number we also trying to find such applications of it.

Keywords - Dominating set, Minimal dominating set, Minimum dominating set, Domination number, perfect dominating set, Minimum perfect dominating set, Minimal perfect dominating set, perfect domination number.

1. Introduction

Here first we define the necessary terms those are mentioned as keywords, also give an example of each if necessary.

Definition 1.1: Let G be a graph and $V(G)$ be the set of all vertices of graph G and S be the subset of $V(G)$ then the set S is said to be dominating set of the graph G if and only if for any $w \in V(G) \setminus S$. It means that we can find at least one vertex $v \in S$ such that w is adjacent to v. For example consider the following graph G, according to the definition of dominating set; the set $S = \{v_1, v_4\}$ is a dominating set. [1] 16-28

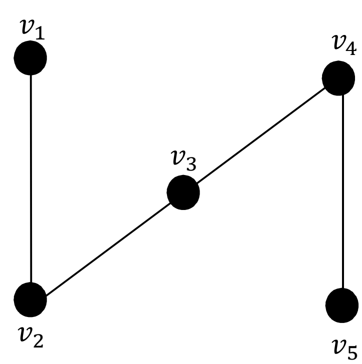

Figure 1: Dominating Set

Definition 1.2: Assume that G be a graph. A dominating set S in the graph G is called minimal dominating set in the graph G if and only if for any $v \in S,\ S \setminus v$ is not a dominating set in the graph G. [1] 16-28. For example consider the following graph G, in which the set $S = \{v_1, v_4\}$ is a minimal dominating set.

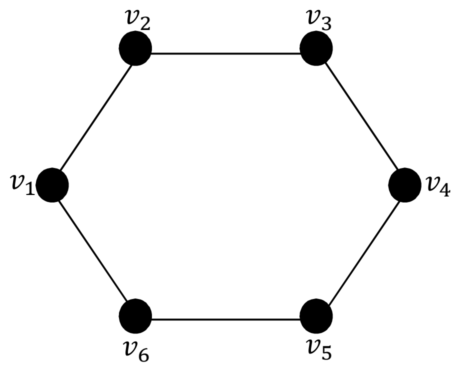

Figure 2: Minimal Dominating Set

Definition 1.3: Let G be a graph. A dominating set S in G with minimum cardinality is called a minimum dominating set. A minimum dominating set in graph G is also called a $\gamma - set$ in the graph G. [1] 16-28. For example consider a following graph G in which according to the definition of minimum dominating set, the set $S = \{v_1\}$ is a minimum dominating set.

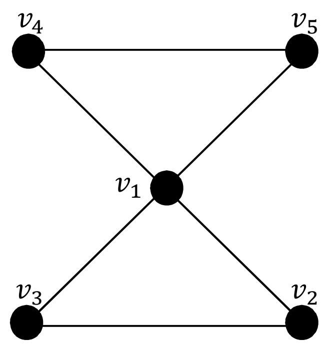

Figure 3: Minimum Dominating Set

Definition 1.4: Consider G be and S being a minimum dominating set in G then $|S|$ means cardinality of set S, is called the domination number of the graph G and it is denoted by $\gamma(G)$. [1] 16-28. For example consider the following graph G, in which according to the definition of domination number, the set $S = \{v_1\}$ is a minimum dominating set and that cardinality is one therefore the domination number of the graph G is equal to 1 means $\gamma(G) = 1$. [1] 16-28.

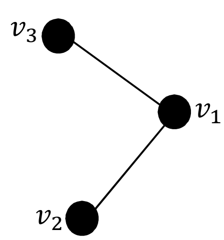

Figure 4: Domination Number

Definition 1.5: A copy graph $G = (V, E)$ is denoted by G' with set of vertices and edges are at it is. For example let us observe following graph $G = C_5$, cycle graph with five vertices then their copy of a graph $G^{'} = c_5$ showing in the figures 5 and 6. [1] 16-28.

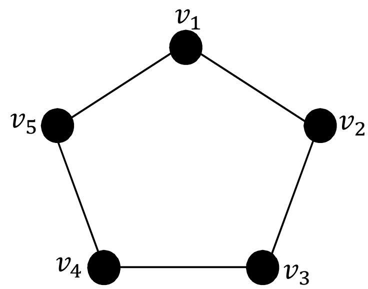

Figure 5: Graph $G = cycle\ C_5$

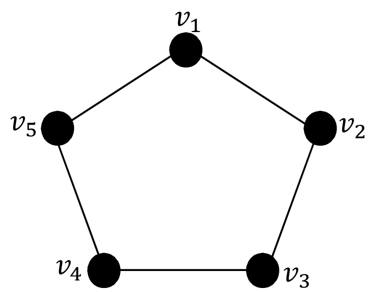

Figure 6: Graph $G' = copy\ of\ a\ graph\ G, cycle\ C_5$

Definition 1.6: A subset S of $V(G)$ is said to be a perfect dominating set if for each vertex v is adjacent to exactly one vertex of S. For example consider the following figure 7, a path graph with for vertices, P_4 in which the set $S_1 = \{v_2, v_3\}$ and $S_2 = \{v_1, v_4\}$ are perfect dominating sets of graph G. It may be noted that if G is a graph then $V(G)$ is always a perfect dominating set of G. [2] 189-239

Figure 7: Graph $G = P_4$, a path graph with four vertices

Definition 1.7: A perfect dominating set S of G is said to be minimal perfect dominating set if each vertex v in S, $S \setminus v$ is not a perfect dominating set. It may be noted that it is not necessary that a proper subset of minimal perfect dominating set is not perfect dominating set. For example consider a following figure 8, a graph $G = C_6$ cycle graph with six vertices then obliviously $V(G)$ is a minimal perfect dominating set of C_6. However the set $S = \{v_1, v_4\}$ is a proper subset of $V(G)$ and is a perfect dominating set of $G = C_6$. [2] 189-239

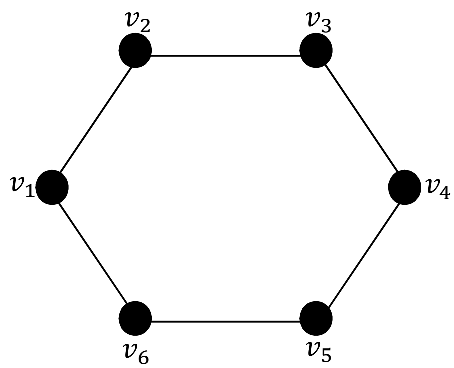

Figure 8: Graph $G = C_6$, a cycle graph with six vertices

Definition 1.9: A perfect dominating set with smallest cardinality is called minimum perfect dominating set and it is also called $\gamma_{pf} - set$. [2] 189-239

Definition 1.10: A cardinality of minimum perfect dominating set is called perfect domination number of the graph G and it is denoted by $\gamma_{pf}(G)$. In the above figure 8, we observe that the perfect domination number of cycle graph C_6 is 2 means $\gamma_{pf}(C_6) = 2$. Moreover consider the following figure 9, a graph $G = P_3$ a path graph with three vertices. [2] 189-239

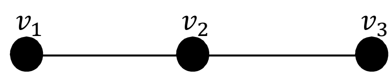

Figure 8: Graph $G = P_3$, a path graph with three vertices

Definition 1.11: The corona product of two graphs G_1 and G_2 is the graph denoted by $G_1 \odot G_2$, is the graph obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 such that i^{th} vertex of the copy of G_1 is adjacent to each vertex of i^{th} copy of G_2. For example consider the following graphs $G_1 = C_3$ cycle graph with three vertices and $G_2 = P_3$ path graph with three vertices showing in the following figures 9 and 10.

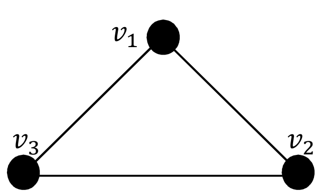

Figure 9: $G = C_3$

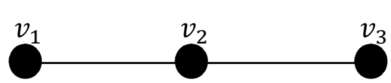

Figure 10: $G = P_3$

Now according to definition of corona product of two graphs we can get a new graph says $G_3 = G_1 \odot G_2$ showing in the following figure 11.

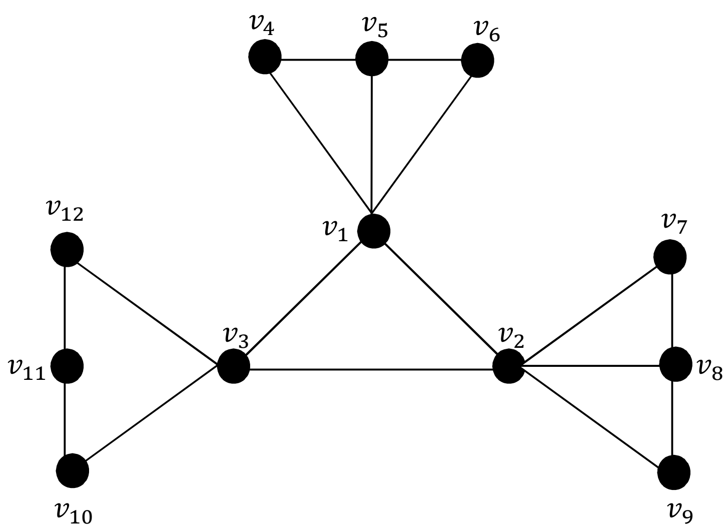

Figure 11: Graph $G_3 = G_1 \odot G_2$

2. Main Results

Let $G = (V, E)$ be any graph where Va set of vertices is and E indicates set of edges. Throughout the result we are consider a cycle graph C_n with n vertices and $n-1$ edges in which we invented some new results regarding the perfect domination number of cycle graph C_n.

2.1 Perfect Domination Number of Cycle graph C_n

Theorem 2.1: Let C_n by any cycle graph with $n \geq 3,$ number of vertices and $n-1$ number of edges where $n \geq 3$ then the perfect domination number of C_n is given by

$$\gamma_{pf}(C_n) = \begin{cases} \frac{2n-1}{5} ; where\ n = 5m-2, m \in N \\ \frac{2(n+1)}{5} ; where\ n = 5m-1, m \in N \\ \frac{2n}{5} ; where\ n = 5m, m \in N \\ \frac{2(n-1)}{5} ; where\ n = 5m+1, m \in N \\ \frac{2n+1}{5} ; where\ n = 5m+2, m \in N \end{cases}$$

Where N indicates the set of natural numbers.

Proof: Here we are proving this theorem by using Principle of Mathematical Indication theory. Now first we are trying to prove that the given result in the hypothesis is true for $n = 3$.

Case I: Let $n = 3$, indicates a cycle graph with three vertices i.e. C_3. For $n = 3$, the suitable value of $m = 1$ follows the $\gamma_{pf}(C_n) = \frac{2n-1}{5}$; $where\ n = 5m - 2, m = 1 \in N$.

$$\Rightarrow m = 1\ follows\ n = 3\ therefore\ \gamma_{pf}(C_3) = \frac{2(3) - 1}{5} = \frac{5}{5} = 1.$$

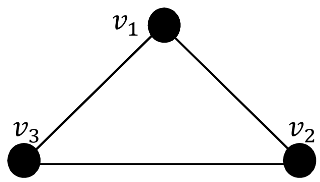

Figure 12: $G = C_3$

According to definition of perfect domination number it is clear that $\gamma_{pf}(C_3) = 1$.
So it is clear that the above theorem is true for $n = 3$.

Case II: Assume that above theorem is true for $n = k$. We have

$$\gamma_{pf}(C_k) = \begin{cases} \frac{2k-1}{5}; where\ k = 5m - 2, m \in N \ \ldots\ldots \ (1) \\ \frac{2(k+1)}{5}; where\ k = 5m - 1, m \in N \ldots\ldots (2) \\ \frac{2k}{5}; where\ k = 5m, m \in N \ \ldots\ldots\ldots\ldots\ldots\ldots \ (3) \\ \frac{2(k-1)}{5}; where\ k = 5m + 1, m \in N \ldots\ldots \ (4) \\ \frac{2k+1}{5}; where\ k = 5m + 2, m \in N \ \ldots\ldots\ldots (5) \end{cases}$$

Case III: Let we can try to prove that above theorem is true for $n = k + 1$.

Claim:

$$\gamma_{pf}(C_{k+1}) = \begin{cases} \frac{2k+1}{5}; where\ k + 1 = 5m - 2, m \in N \\ \frac{2(k+2)}{5}; where\ k + 1 = 5m - 1, m \in N \\ \frac{2(k+1)}{5}; where\ k + 1 = 5m, m \in N \\ \frac{2(k)}{5}; where\ k + 1 = 5m + 1, m \in N \\ \frac{2k+3}{5}; where\ k + 1 = 5m + 2, m \in N \end{cases}$$

To prove: (1) $\gamma_{pf}(C_{k+1}) = \frac{2(k+1)-1}{5} = \frac{2k+1}{5}$; $where\ k + 1 = 5m - 2, m \in N$

Assume that $\lambda = k + 1$,

$\Rightarrow \lambda = 5m - 2$ follows the above result …. (1) which is true for every $k = 5m - 2$

$$(1) \Rightarrow \gamma_{pf}(C_k) = \frac{2k-1}{5}; where\ k = 5m - 2, m \in N$$

$\Rightarrow \gamma_{pf}(C_\lambda) = \frac{2\lambda-1}{5}; where\ \lambda = 5m - 2, m \in N$

But $\lambda = k + 1$ than we get

$$\therefore \gamma_{pf}(C_{k+1}) = \frac{2(k+1)-1}{5} = \frac{2k+1}{5}\ ; where\ k + 1 = 5m - 2, m \in N$$

Hence prove the result.

To prove: (2) $\gamma_{pf}(C_{k+1}) = \frac{2(k+1+1)}{5} = \frac{2(k+2)}{5}; where\ k + 1 = 5m - 1, m \in N$

Let $\alpha = k + 1,$
$\Rightarrow \alpha = 5m - 1$ follows the above result ….(2) which is true for every $k = 5m - 1.$

$$(2) \Rightarrow \gamma_{pf}(C_k) = \frac{2(k+1)}{5}; where\ k = 5m - 1, m \in N$$

$\Rightarrow \gamma_{pf}(C_\alpha) = \frac{2(\alpha+1)}{5}; where\ \alpha = 5m - 1, m \in N$

But $\alpha = k + 1$ than we get

$$\gamma_{pf}(C_{k+1}) = \frac{2(k+1+1)}{5} = \frac{2(k+2)}{5}; where\ k + 1 = 5m - 1, m \in N$$

Hence prove the result.

To prove: (3) $\gamma_{pf}(C_{k+1}) = \frac{2(k+1)}{5}; where\ k + 1 = 5m, m \in N$

Let $\beta = k + 1,$

$\Rightarrow \beta = 5m, m \in N$ follow the above result … (3) which is true for every k

$$(3) \Rightarrow \gamma_{pf}(C_k) = \frac{2k}{5}; where\ k = 5m, m \in N$$

$$\Rightarrow \gamma_{pf}(C_\beta) = \frac{2\beta}{5}; where\ \beta = 5m, m \in N$$

But $\beta = k + 1$ than we get

$$\gamma_{pf}(C_{k+1}) = \frac{2(k+1)}{5}; where\ k+1 = 5m, m \in N$$

Hence prove the result.

To prove: (4) $\gamma_{pf}(C_{k+1}) = \frac{2(k)}{5}; where\ k+1 = 5m+1, m \in N$

Let $\delta = k + 1,$

$\Rightarrow \delta = 5m + 1, m \in N$ follows the above result … (4) which is true for every k

$(4) \Rightarrow \gamma_{pf}(C_k) = \frac{2(k-1)}{5}; where\ k = 5m+1, m \in N$

$$\Rightarrow \gamma_{pf}(C_\delta) = \frac{2(\delta-1)}{5}; where\ \delta = 5m+1, m \in N$$

But $\delta = k + 1$ than we get

$$\gamma_{pf}(C_{k+1}) = \frac{2(k)}{5}; where\ k+1 = 5m+1, m \in N$$

Hence prove the result.

To prove: (5) $\gamma_{pf}(C_{k+1}) = \frac{2k+3}{5}; where\ k+1 = 5m+2, m \in N$

Let $\theta = k + 1,$

$\Rightarrow \theta = 5m + 2, m \in N$ follows the above result … (5) which is true for every k

$$(5) \Rightarrow \gamma_{pf}(C_k) = \frac{2k+1}{5}; where\ k = 5m+2, m \in N$$

$$\Rightarrow \gamma_{pf}(C_\theta) = \frac{2\theta+1}{5}; where\ \theta = 5m+2, m \in N$$

But $\theta = k + 1$ than we get

$$\gamma_{pf}(C_{k+1}) = \frac{2k+3}{5}; where\ k+1 = 5m+2, m \in N$$

Hence prove the result.

So it is clarify that above theorem is passes through all the testing parameters of mathematical induction therefore we say that above theorem is true for $n = k + 1$. Now verify the above theorem by taking an example of cycle graph with 10 vertices i.e. Graph $G = C_{10}$, for C_9 we can try to obtain $\gamma_{pf}(C_{10})$. Now consider the following figure 13.

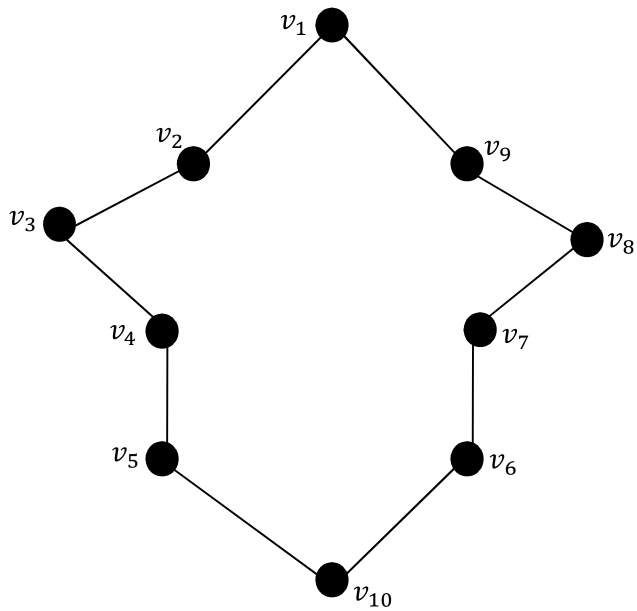

Figure 13: $G = C_{10}$, a cycle graph with 10 vertices

Now according to the definition of perfect domination number, the vertex set $V(C_{10})$ in which each vertex of it, is adjacent to exactly one vertex of a set $S = \{v_1, v_4, v_7, v_{10}\}$ therefore a set S is a minimum perfect dominating set with cardinality 4. Therefore $\gamma_{pf}(C_{10}) = 4$.

Now according to our theorem for $n = 10$, the suitable case is $n = 5m, m = 2 \in N$ and hence $\gamma_{pf}(C_n) = \frac{2n}{5}$ follows for $n = 10$ we get $\gamma_{pf}(C_{10}) = \frac{2(10)}{5} = \frac{20}{5} = 4$.

Now verify the above theorem by taking an example of cycle graph with 13 vertices that is graph $G = C_{13}$ we can try to obtain $\gamma_{pf}(C_{13})$. Now consider the following figure 14.

According to definition of perfect domination number, the vertex set $V(C_{13})$ in which each vertex of it is adjacent to exactly one vertex of a set $S = \{v_1, v_4, v_7, v_8, v_{11}\}$ therefore a set S is a minimum perfect dominating set with cardinality 5. Therefore $\gamma_{pf}(C_{13}) = 5$.

Now according to above mentioned theorem 2.1 we take $n = 13$, is suitable for the case $n = 5m - 2, m = 3 \in N$ follows $\gamma_{pf}(C_n) = \frac{2n-1}{5}$ and hence $\gamma_{pf}(C_{13}) = \frac{2(13)-1}{5} = \frac{25}{5} = 5$.

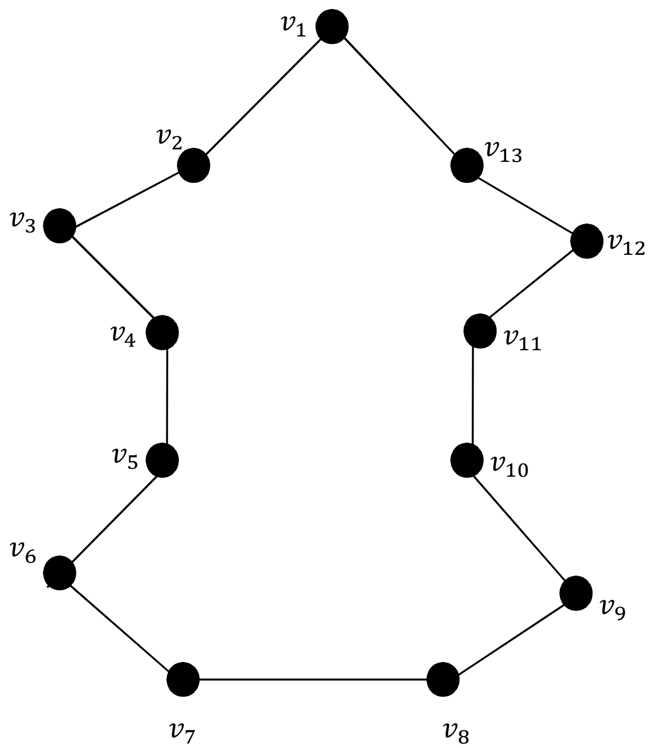

Figure 14: $G = C_{13}$; a cycle graph with 13 vertices

3. Conclusion

Here we obtained the perfect domination number of cycle graph with $n -$ vertices for some $n \in N$. It is useful for measuring shortest distances form source vertex (node) to destination vertex (node) without using any algorithmic process of complex mathematical calculations.

There is also needful for creating a networks and its efficiency verification. The perfect domination number is widely used in coding theory, by using this concept the computer programmer will able to decode some complicated conjectures.

4. References

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