On Kasaj Generalized Closed Sets in Kasaj Topological Spaces

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Abstract: We introduced Kasaj topological spaces which is a partial extension of Micro topological space which is introduced by S. Chandrasekar. We also analyzed basic properties of some weak open sets in Kasaj topological spaces. In this paper, we shall introduce Kasaj-generalized-closed (briefly, KSg-closed) set and study their basic properties in Kasaj topological spaces.

***IndexTerms* - Kasaj topological space, Kasaj generalized closed sets.**

I. Introduction and Preliminary

In year 1970, Levine **[2]** introduced the class of generalized closed set which is super class of closed set. This was introduced as generalization of closed set in Topological space and new results were proved. M.L. Thivagar et. al. **[1]** introduced Nano Topological spaces with respect to a subset X of a universal set Ω which is defined in terms of lower and upper approximation of X. Bhuvaneswari and K.M.Gnanapriya **[7]** studied nano generalised closed set in Nano topological spaces. S. Chandrasekar **[8]** introduced a new topology namely, Micro topology which is an extension of nano topological space. For this extension he has used Levine's simple extension method. In this year, we **[4]** introduced partial extension of Micro topological space namely Kasaj topological spaces. We also defined and studied properties of Kasaj-pre-open **[4]**, Kasaj-semi-open **[4]**, Kasaj-α-open **[5]**, Kasaj-β-open **[5]**, Kasaj-b-open **[6]** and Kasaj-regular-open **[6]** set. In this paper we shall define Kasaj-generalised-closed sets in Kasaj topological spaces and also study their properties.

Definition: 1[2] A subset P of a topological space (X, τ) is called a generalized closed set (briefly g-closed) if $Cl(P) \subseteq V$ whenever $P \subseteq V$ and V is open set in (X, τ).

II. Nano Topological Spaces

Definition: 2[1] Let Ω be a non-empty Universal set and $\mathfrak{R}$ be an equivalence relation on Ω and it is named as *the indiscernibility relation*. As $\mathfrak{R}$ is an equivalence relation, we get disjoint partition and each equivalence class is indiscernible with one another. The pair (Ω, $\mathfrak{R}$) is called as *approximation space*. Let $X \subset \Omega$.

- *The lower approximation* of X with respect to $\mathfrak{R}$ is denoted by $\mathscr{L}_{\mathfrak{R}}(X)$ and is defined by $\mathscr{L}_{\mathfrak{R}}(X) = \cup_{x\in\Omega}\{P(x) : P(x) \subset X\}$ where P(x) denotes the equivalence relation which contains $x\in\Omega$.
- The upper approximation of X with respect to $\mathfrak{R}$ is denoted by $\mathbf{U}_{\mathfrak{R}}(X)$ and is defined by $\mathbf{U}_{\mathfrak{R}}(X) = \cup_{x\in\Omega}\{P(x) : P(x) \cap X \neq \varnothing\}$ where P(x) denotes the equivalence relation which contains $x\in\Omega$.
- The boundary region of X with respect to $\mathfrak{R}$ is denoted by $\mathscr{B}_{\mathfrak{R}}(X)$ and is defined by $\mathscr{B}_{\mathfrak{R}}(X) = \mathbf{U}_{\mathfrak{R}}(X) - \mathscr{L}_{\mathfrak{R}}(X)$.

Definition: 3[1] Let Ω be an universal set. $\mathfrak{R}$ be an equivalence relation on Ω, $X \subset \Omega$ and $\tau_{\mathfrak{R}}(X) = \{ \Omega, \varnothing, \mathscr{L}_{\mathfrak{R}}(X), \mathbf{U}_{\mathfrak{R}}(X), \mathscr{B}_{\mathfrak{R}}(X)\}$ which satisfies the following axioms.

- $\Omega, \varnothing \in \tau_{\mathfrak{R}}(X)$.
- The union of elements of any subcollection of $\tau_{\mathfrak{R}}(X)$ is in $\tau_{\mathfrak{R}}(X)$.
- The intersection of any finite subcollection of elements of $\tau_{\mathfrak{R}}(X)$ is in $\tau_{\mathfrak{R}}(X)$.

Then (Ω, $\tau_{\mathfrak{R}}(X)$) is called *nano topological space*. The members of $\tau_{\mathfrak{R}}(X)$ are called *nano open sets*.

III. Micro Topological Spaces

Definition: 4 [8] Let (Ω, $\tau_{\mathfrak{R}}(X)$) be a nano topological space and *micro topology* is defined by

$$\mu_{\mathfrak{R}}(X) = \{K \cup (K' \cap S) : K, K' \in \tau_{\mathfrak{R}}(X), \text{fixed } S \notin \tau_{\mathfrak{R}}(X)\}.$$

Definition: 5 [8] *The Micro topology* $\mu_{\Re}(X)$ satisfies the following postulates :

- $\Omega, \varnothing \in \mu_{\Re}(X)$.
- The union of elements of any subcollection of $\mu_{\Re}(X)$ is in $\mu_{\Re}(X)$.
- The intersection of any finite subcollection of elements of $\mu_{\Re}(X)$ is in $\mu_{\Re}(X)$.

Then $(\Omega, \tau_{\Re}(X), \mu_{\Re}(X))$ is called *Micro topological spaces* and the members of $\mu_{\Re}(X)$ are called *Micro open sets (μ-open sets)* and the complement of a μ-open set is called a μ-closed set.

IV. KASAJ TOPOLOGICAL SPACE

Definition: 6 [4] Let $(\Omega, \tau_{\Re}(X))$ be a nano topological space. Then *Kasaj topology* is defined by

$$KS_{\Re}(X) = \{(K \cap S) \cup (K' \cap S') : K, K' \in \tau_{\Re}(X), \text{ fixed } S, S' \notin \tau_{\Re}(X), S \cup S' = \Omega\}.$$

Definition: 7[4] The *Kasaj topology* $KS_{\Re}(X)$ satisfies the following postulates :

- $\Omega, \emptyset \in KS_{\Re}(X)$.
- The union of elements of any subcollection of $KS_{\Re}(X)$ is in $KS_{\Re}(X)$.
- The intersection of any finite subcollection of elements of $KS_{\Re}(X)$ is in $KS_{\Re}(X)$.

Then $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ is called *Kasaj topological space* and the members of $KS_{\Re}(X)$ are called *Kasaj open sets (KS-open sets)* and the complement of a *Kasaj-open set* is called a *Kasaj-closed (KS-closed) set* and the collection of all Kasaj-closed sets is denoted by KSCL(X).

Definition: 8[4] The Kasaj closure and the Kasaj interior of a set **P** is denoted by $KS_{cl}(P)$ and $KS_{int}(P)$, respectively. It is defined by $KS_{cl}(P) = \cap\{Q : P \subset Q, Q \text{ is KS-closed}\}$ and $KS_{int}(P) = \cup\{Q : Q \subset P, Q \text{ is KS-open}\}$.

Remark: 9[4]

- $KS_{int}(P)$ is the largest KS-open set contained in P.
- $KS_{cl}(P)$ is the smallest KS-closed set containing P.

Definition:10[4] For any two subsets P, Q of Ω in a *Kasaj topological space* $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$,

- P is a Kasaj-closed set if and only if $KS_{cl}(P) = P$.
- P is a Kasaj-open set if and only if $KS_{int}(P) = P$.
- If $P \subset Q$, then $KS_{int}(P) \subset KS_{int}(Q)$ and $KS_{cl}(P) \subset KS_{cl}(Q)$.
- $KS_{cl}(KS_{cl}(P)) = KS_{cl}(P)$ and $KS_{int}(KS_{int}(P)) = KS_{int}(P)$.
- $KS_{cl}(P) \cup KS_{cl}(Q) \subset KS_{cl}(P \cup Q)$.
- $KS_{int}(P) \cup KS_{int}(Q) \subset KS_{int}(P \cup Q)$.
- $KS_{cl}(P \cap Q) \subset KS_{cl}(P) \cap KS_{cl}(Q)$.
- $KS_{int}(P \cap Q) \subset KS_{int}(P) \cap KS_{int}(Q)$.
- $KS_{cl}(P) = [KS_{int}(P)]^c$.
- $KS_{int}(P) = [KS_{cl}(P)]^c$.

V. KASAJ-GENERALIZED-CLOSED SETS

In this section, we define Kasaj-generalised-closed sets and study their properties.

Definition: 11 A subset P of X in $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ is called Kasaj-generalised closed (briefly, KSg-closed) set if $KS_{cl}(P) \subseteq V$ whenever $P \subseteq V$ and V is Kasaj-open set in $KS_{\Re}(X)$.

A subset P is called KSg-open in $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ if P^c is KSg-closed set. The collection of all KSg-closed sets is denoted by $KSgC(\Omega,X)$ and The collection of all KSg-open sets is denoted by $KSgO(\Omega,X)$.

Definition: 12 A Kasaj-generalized closure of P is defined as the intersection of all KSg-closed sets containing P. It is denoted by $KSg_{cl}(P)$.

Example: 13 Let $\Omega = \{\lambda, \theta, \pi, \upsilon, \kappa\}$ with $\Omega/\Re = \{\{\lambda\}, \{\theta, \pi, \upsilon\}, \{\kappa\}\}$ and $X = \{\upsilon, \kappa\} \subseteq \Omega$. Then $\tau_{\Re}(X) = \{\emptyset, \Omega, \{\kappa\}, \{\theta, \pi, \upsilon\}, \{\theta, \pi, \upsilon, \kappa\}\}$. If we consider $S = \{\lambda,\theta, \upsilon\}$ and $S' = \{\pi, \kappa\}$, then

- $KS_{\Re}(X) = \{\emptyset, \{\pi\}, \{\kappa\}, \{\pi, \kappa\}, \{\theta, \kappa\}, \{\lambda, \theta, \upsilon\}, \{\theta, \upsilon, \kappa\}, \{\theta, \pi, \upsilon\}, \{\lambda, \theta, \upsilon, \kappa\}, \{\lambda, \theta, \pi, \upsilon\}, \{\theta, \pi, \upsilon, \kappa\}, \Omega\}$.
- $KSgC(\Omega, X) = \{\emptyset, \{\lambda\}, \{\pi\}, \{\kappa\}, \{\lambda, \theta\}, \{\lambda, \pi\}, \{\lambda, \upsilon\},\{\lambda, \kappa\}, \{\pi, \kappa\}, \{\lambda, \theta, \pi\}, \{\lambda, \theta, \upsilon\}, \{\lambda, \theta, \kappa\}, \{\lambda, \pi, \upsilon\}, \{ \lambda, \pi, \kappa\}, \{\lambda, \upsilon, \kappa\}, \{\lambda, \theta, \pi, \upsilon\}, \{\lambda, \theta, \pi, \kappa\}, \{\lambda, \theta, \upsilon, \kappa\}, \{\lambda, \pi, \upsilon, \kappa\}, \Omega\}$.
- $KSgO(\Omega, X) = \{\emptyset, \{\theta\}, \{\pi\}, \{\upsilon\}, \{\kappa\}, \{\theta, \pi\}, \{\theta, \upsilon\}, \{\theta, \kappa\},\{\pi, \upsilon\}, \{\pi, \kappa\}, \{\upsilon, \kappa\}, \{\lambda, \theta, \upsilon\}, \{\theta, \pi, \upsilon\}, \{\theta, \pi, \kappa\}, \{\theta, \upsilon, \kappa\},\{\pi, \upsilon, \kappa\}, \{\lambda, \theta, \pi, \upsilon\}, \{\lambda, \theta, \upsilon, \kappa\}, \{\theta, \pi, \upsilon, \kappa\}, \Omega\}$.

Remark: 14

- KSg_{cl} (P) is the smallest KSg-closed set containing P. In general, $P \subseteq KSg_{cl}$ (P).
- If P is KSg-closed set, then $P = KSg_{cl}$ (P).
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Definition: 15 A Kasaj-generalised interior of P is defined as the union of all KSg-open sets contained in P. It is denoted by $KSg_{int}(P)$.

- KSg_{int} (P) is the largest KSg-open set contained in P. In general, KSg_{int} (P) $\subseteq$ P.
- If P is KSg-open set, then $P = KSg_{int}$ (P).

Theorem: 16 $KSCL(\Omega, X) \subseteq KSgC(\Omega, X)$.

Proof: Let $P \in KSCL(\Omega, X)$, i.e., KS_{cl} (P) = P and let $P \subseteq V$ and $V \in KS_{\Re}(X)$. Then KS_{cl} (P) = $P \subseteq V$ whenever $P \subseteq V$ and $V \in KS_{\Re}(X)$. Thus, P is KSg-closed set, i.e., $P \in KSgC(\Omega, X)$.

Remark: 17 In general, $KSgC(\Omega, X) \not\subset KSCL(\Omega, X)$. $KSgC(\Omega, X)$. In Example 13, one can see that $\{\lambda, \theta\} \in KSgC(\Omega, X)$ but $\{\lambda, \theta\} \notin KSCL(\Omega, X)$. From the above Theorem, we can see that $KS_{\Re}(X) \subseteq KSgO(\Omega, X)$.

Theorem: 18

1. $KS_{int}(\Omega \backslash P) = \Omega \setminus KS_{cl}(P)$.
2. $KS_{cl}(\Omega \backslash P) = \Omega \setminus KS_{int}(P)$.

Theorem: 19 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be Kasaj topological space. Then $P \in KSgO(\Omega, X)$ if and only if $F \subseteq KS_{int}(P)$ whenever $F \subseteq P$ and $F \in KSCL(\Omega, X)$.

Proof: $(\Rightarrow)$ Assume that $F \subseteq KS_{int}(P)$ where F is KS-closed set with $F \subseteq P$. Let $P^c \subseteq U$ where $U \in KS_{\Re}(X)$. Then $U^c \subseteq P$ and $U^c \in KSCL(\Omega, X)$. By our assumption $U^c \subseteq KS_{int}$ (P), which implies that $(KS_{int}(P))^c \subseteq U$. By the Theorem 18, we have $(KS_{int}(P))^c = KS_{cl}(P^c)$. So, $KS_{cl}(P^c) \subseteq U$. Thus, P^c is KSg-closed set, i.e., $P \in KSgO(\Omega, X)$.

$(\Leftarrow)$ Conversely, assume that $P \in KSgO(\Omega, X)$. i.e., $P^c \in KSCL(\Omega, X)$. i.e., $KS_{cl}(P^c) \subseteq V$ whenever $P^c \subseteq V$ and $V \in KS_{\Re}(X)$. Now, let $F \in KS_{\Re}(X)$ with $F \subseteq P$. Then $F^c \in KS_{\Re}(X)$ with $P^c \subseteq F^c$. Since $P^c \in KSCL(\Omega, X)$, $KS_{cl}(P^c) \subseteq F^c$. Then by Theorem 18, $F \subseteq (KS_{cl}(P^c))^c = KS_{int}(P)$.

Theorem: 20

I. If $P, Q \in KSgC(\Omega, X)$, then $P \cap Q \in KSgC(\Omega, X)$.

II. If P, Q ∈ KSgO(Ω, X), then P ∪ Q ∈ KSgO(Ω, X).

Proof(I): Let P, Q ∈ KSgC(Ω, X). Then $KS_{cl}(P) \subseteq V$ where $P \subseteq V$ and $V \in KS_{\Re}(X)$ and $KS_{cl}(Q) \subseteq V$ where $Q \subseteq V$ and $V \in KS_{\Re}(X)$. Since P, Q ⊆ V, P ∩ Q ⊆ V and $V \in KS_{\Re}(X)$. Then $KS_{cl}(P \cap Q) \subseteq KS_{cl}(P) \cap KS_{cl}(Q) \subseteq V$ which implies that P ∩ Q ∈ KSgC(Ω, X).

Proof(II): Let P, Q ∈ KSgO(Ω, X). Then $V \subseteq KS_{int}(P)$ where $V \subseteq P$ and V ∈ KSCL(Ω, X) and $V \subseteq KS_{int}(Q)$ where $V \subseteq Q$ and V ∈ KSCL(Ω, X). Since V ⊆ P and V ⊆ Q, V ⊆ P ∪ Q and V ∈ KSCL(Ω, X). Then $V \subseteq KS_{int}(P) \cup KS_{int}(Q) \subseteq KS_{int}(P \cup Q)$. which implies that P ∪ Q ∈ KSgO(Ω, X).

Proposition: 21 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be Kasaj topological space. Then $KSgC(\Omega, X) \cap KS_{\Re}(X) \subseteq KSCL(\Omega, X)$.

Proof: Assume that $P \in KSgC(\Omega, X) \cap KS_{\Re}(X)$, i.e., $KS_{int}(P) = P$ and $KS_{cl}(P) \subseteq V$ whenever $P \subseteq V$ and $V \in KS_{\Re}(X)$. Then, in particular, $KS_{cl}(P) \subseteq P$. Hence $KS_{cl}(P) = P$. i.e., P ∈ KSCL(Ω, X).

Theorem: 22 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be Kasaj topological space and let P ⊆ Ω. Then

- $KS_{int}(P) \subseteq KSg_{int}(P)$.
- $KSg_{cl}(P) \subseteq KS_{cl}(P)$.

Definition: 23 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be a Kasaj topological space. Then P is said to be Kasaj-generalized neighbourhood (briefly, KSg-nbhd) of a point x of Ω if there exists KSg-open set F containing x such that x ∈ F ⊆ P.

Remark: 24 Every KSg-nbhd is a KSg-open set.

Theorem: 25 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be a Kasaj topological space and let P ⊆ Ω. Then P ∈ KSgO(Ω, X) if and only if it is a KSg-nbhd of each of its points.

Proof: (⇒) Assume that P ∈ KSgO(Ω, X) and x ∈ P, then x ∈ P ⊆ P. Hence P is a KSg-nbhd of each of its points.

(⇐) Conversely, assume that P is a KSg-nbhd of each of its points. Then P is KSg-open set containing each of its points. So, P is KSg-open set.

Theorem: 26 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be a Kasaj topological space and P ⊆ Ω. Then P ∈ KSgC(Ω, X) if and only if there does not exist non-empty F ∈ KSCL(Ω, X) such that $F \subseteq KS_{cl}(P)\backslash P$.

Proof: (⇒) Let P ∈ KSgC(Ω, X) and let F ∈ KSCL(Ω, X) such that $F \subseteq KS_{cl}(P)\backslash P$. Then $P \subseteq F^c$. As F ∈ KSCL(Ω, X), $F^c \in KS_{\Re}(X)$. Since P ∈ KSgC(Ω, X), $KS_{cl}(P) \subseteq F^c$ or $F \subseteq (KS_{cl}(P))^c$. One can also observe that $F \subseteq KS_{cl}(P)\backslash P \subseteq KS_{cl}(P)$, which implies that $F \subseteq KS_{cl}(P) \cap (KS_{cl}(P))^c = \varnothing$. So, F = ∅.

(⇐) Assume that there does not exist non-empty F ∈ KSCL(Ω, X) such that $F \subseteq KS_{cl}(P)\backslash P$. Let P ⊆ F and $F \in KS_{\Re}(X)$. We prove this by contradiction. Suppose that $KS_{cl}(P) \not\subset F$, then $KS_{cl}(P) \cap F^c \neq \varnothing$. Since $KS_{cl}(P) \in KSCL(\Omega, X)$ and $F^c \in KSCL(\Omega, X)$. Therefore, $KS_{cl}(P) \cap F^c \in KSCL(\Omega, X)$. But $\varnothing \neq KS_{cl}(P) \cap F^c \subseteq KS_{cl}(P) \cap P^c = KS_{cl}(P)\backslash P$, which is contradiction to the given assumption. Hence, $KS_{cl}(P) \subseteq F$. i.e., P ∈ KSgC(Ω, X).

Theorem: 27 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be a Kasaj topological space and P ⊆ Ω. Then P ∈ KSgC(Ω, X) if and only if P = F\N where F ∈ KSCL(Ω, X) and N which does not contain any non-empty KS-closed set.

Proof: (⇒) Assume that $P \in KSgC(\Omega, X)$. Then if we consider $F = KS_{cl}(P)$ and $N = KS_{cl}(P)\backslash P$. Then $P = F\backslash N$. By Theorem 26, N does not contain nonempty KS-closed set.

(⇐)Conversely, assume that $P = F\backslash N$, where $F \in KSCL(\Omega, X)$ and N does not contain any nonempty KS-closed set. Let $P \subseteq U$ and $U \in KS_{\Re}(X)$. Then $F \cap U^c \in KSCL(\Omega, X)$. Since $P = F\backslash N$ (i.e., $N = F\backslash P$) and $P \subseteq U$ (i.e. $U^c \subseteq P^c$), $F \cap U^c \subseteq F \cap P^c = F\backslash P = N$. By the given hypothesis $F \cap U^c = \varnothing$. i.e., $F \subseteq U$. Thus $P \subseteq F\backslash N \subseteq F$, which implies that $KS_{cl}(P) \subseteq KS_{cl}(F) = F \subseteq U$. Hence, $P \in KSgC(\Omega, X)$.

Theorem: 28 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be Kasaj topological space and let $P, Q \subseteq \Omega$ be such that $P \subseteq Q \subseteq KS_{cl}(P)$. Then if $P \in KSgC(\Omega, X)$, then $Q \in KSgC(\Omega, X)$.

Proof: Assume that $P \in KSgC(\Omega, X)$. i.e., $KS_{cl}(P) \subseteq V$ where $P \subseteq V$ and $V \in KS_{\Re}(X)$. Since, $Q \subseteq KS_{cl}(P)$, then

$$KS_{cl}(Q) \subseteq KS_{cl}(KS_{cl}(P)) = KS_{cl}(P). \quad \ldots(1)$$

Now let $Q \subseteq V'$ and $V' \in KS_{\Re}(X)$. Since $P \subseteq Q \subseteq KS_{cl}(P)$ and $P \in KSgC(\Omega, X)$, $KS_{cl}(P) \subseteq V'$ Then by equation (1), we get $KS_{cl}(Q) \subseteq V'$. Hence $Q \in KSgC(\Omega, X)$.

Theorem: 29 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be Kasaj topological space and $P \subseteq \Omega$. Then $P \in KSgO(\Omega, X)$ if and only if $F = \Omega$ whenever $F \in KS_{\Re}(X)$ and $KS_{int}(P) \cup P^c \subseteq F$.

(⇒) Assume that $P \in KSgO(\Omega, X)$. $KS_{cl}(P^c) \subseteq V$ where $P^c \subseteq V$ and $V \in KS_{\Re}(X)$. Let $F \in KS_{\Re}(X)$ with $KS_{int}(P) \cup P^c \subseteq F$. Then, $F^c \subseteq (KS_{int}(P) \cup P^c)^c = (KS_{int}(P))^c \cap (P^c)^c = (KS_{int}(P))^c\backslash P^c = KS_{cl}(P^c)\backslash P^c$.

Since $P^c \in KSgC(\Omega, X)$ and $F^c \in KSgC(\Omega, X)$. Then by Theorem 19, $F^c = \varnothing$, i.e., $F = \Omega$.

(⇐) Assume that if $F \in KS_{\Re}(X)$ and $KS_{int}(P) \cup P^c \subseteq F$, then $F = \Omega$. Let $F \in KSgC(\Omega, X)$ with $F \subseteq P$. Then $KS_{int}(P) \cup P^c \subseteq KS_{int}(P) \cup F^c$. Since both $KS_{int}(P)$, $F^c \in KS_{\Re}(X)$, then their union is also in $KS_{\Re}(X)$. By the given hypothesis $KS_{int}(P) \cup F^c = \Omega$, which implies that $F \cap (KS_{int}(P))^c = \varnothing$. i.e., $F \subseteq KS_{int}(P)$. Hence, $P \in KSgO(\Omega, X)$.

Theorem: 30 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be Kasaj topological space and $P \subseteq \Omega$. Then $P \in KSgC(\Omega, X)$ if and only if $KS_{cl}(P)\backslash P \in KSgO(\Omega, X)$.

Proof: (⇒) Assume that $P \in KSgC(\Omega, X)$. Let $F \subseteq KS_{cl}(P)\backslash P$ where $F \in KSCL(\Omega, X)$ with $F \subseteq P$. Then by Theorem 19, $F = \varnothing$. So, we have $\varnothing = F \subseteq KS_{int}(KS_{cl}(P)\backslash P)$. Hence, $KS_{cl}(P)\backslash P \in KSgO(\Omega, X)$.

(⇐)Assume that $KS_{cl}(P)\backslash P \in KSgC(\Omega, X)$. Let $P \subseteq V$ and $V \in KSgO(\Omega, X)$. Then $KS_{cl}(P) \cap V^c \subseteq KS_{cl}(P) \cap P^c = KS_{cl}(P)\backslash P$. Since $KS_{cl}(P)$, $V^c \in KSCL(\Omega, X)$, $KS_{cl}(P) \cap V^c \in KSCL(\Omega, X)$. Since $KS_{cl}(P)\backslash P \in KSgO(\Omega, X)$, $F \subseteq KS_{int}(KS_{int}(P)\backslash P)$ where $F \in KSCL(\Omega, X)$ with $F \subseteq P$. Then in particular $KS_{cl}(P) \cap V^c \subseteq KS_{int}(KS_{cl}(P)\backslash P) = \varnothing$.

$KS_{cl}(P) \cap V^c \subseteq KS_{int}(KS_{cl}(P)\backslash P) = KS_{int}(KS_{cl}(P) \cap P^c) = KS_{int}(KS_{cl}(P)) \cap KS_{int}(P^c) = KS_{int}(KS_{cl}(P)) \cap (KS_{cl}(P))^c = KS_{int}(KS_{cl}(P))\backslash KS_{cl}(P) = \varnothing$.

It follows that $KS_{cl}(P) \subseteq V$. Hence, $P \in KSgC(\Omega, X)$.

Theorem: 31

- $KSg_{int}(P \cap Q) = KSg_{int}(P) \cap KSg_{int}(Q)$.
- $KSg_{int}(P \cup Q) = KSg_{int}(P) \cup KSg_{int}(Q)$.
- $KSg_{cl}(P \cap Q) = KSg_{cl}(P) \cap KSg_{cl}(Q)$.
- $KSg_{cl}(P \cup Q) = KSg_{cl}(P) \cup KSg_{cl}(Q)$.

- If $P \subseteq Q$, then $KSg_{int}(P) \subseteq KSg_{int}(Q)$.
- If $P \subseteq Q$, then $KSg_{cl}(P) \subseteq KSg_{cl}(Q)$.
- $KSg_{cl}(\Omega) = \Omega$.
- $KSg_{int}(\Omega) = \Omega$.
- $KSg_{cl}(\varnothing) = \varnothing$.
- $KSg_{int}(\varnothing) = \varnothing$.
- $KSg_{cl}(KSg_{cl}(P)) = KSg_{cl}(P)$.
- $KSg_{int}(KSg_{int}(P)) = KSg_{int}(P)$.
- $KSg_{int}(P^c) = (KSg_{cl}(P))^c$.
- $KSg_{cl}(P^c) = (KSg_{int}(P))^c$.

Theorem: 32 Let $(\Omega, \tau_{\Re}(X), KS_{\Re}(X))$ be a Kasaj topological space and $P \subseteq \Omega$. Then $x \in KSg_{cl}(P)$ if and only if for any KSg-nbhd N of x, $P \cap N_x \neq \varnothing$.

Proof: ($\Rightarrow$) Assume that $x \in KSg_{cl}(P)$. We prove this by contradiction. So, suppose that N_x be a KSg-nbhd of x such that $P \cap N_x = \varnothing$. Since N_x is KSg-nbhd of x, there exists V_x containing x such that $x \in V_x \subseteq N_x$. So, it follows that $V_x \cap P = \varnothing$. So, $P \subseteq (V_x)^c$. Since $(V_x)^c \in KSgC(\Omega, X)$ containing P, $KSg_{cl}(P) \subseteq (V_x)^c$. Hence $x \in (V_x)^c$. Which is contradiction.

($\Leftarrow$) Conversely, assume that N_x is a KSg-nbhd of x with $P \cap N_x \neq \varnothing$. If possible suppose that $x \notin KSg_{cl}(P)$, then there exists a $F \in KSgC(\Omega, X)$ such that $P \subseteq F$ and $x \notin F$, which implies that $x \in F^c$ and F^c is KSg-nbhd of x. But $P \cap F^c = \varnothing$, which is contradiction. Hence $x \in KSg_{cl}(P)$.

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