Congruent Domination Number of Graphs Obtained by Means of Some Graph Operation

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Abstract— **A dominating set $D \subseteq V(G)$ is said to be a congruent dominating set of a graph G if $\sum_{v\in V(G)} d(v) \equiv 0(\text{mod } \sum_{v\in D} d(v))$. The minimum cardinality of a minimal congruent dominating set of G is called the congruent domination number of G which is denoted by $\gamma_{cd}(G)$. In this paper, we investigate congruent domination number of some graphs obtained by means of some graph operation.**

***Keywords*— Dominating Set, Domination Number, Congruent Dominating Set, Congruent Domination Number**

I. INTRODUCTION

Domination in graphs is one of the concepts in graph theory that has piqued the interest of many researchers due to its potential to solve real-world problems involving communication network design and analysis, as well as defence surveillance. There are numerous domination models available in the literature. [1, 4, 6, 7, 8, 9] provide a concise explanation of dominating sets and related concepts. For standard notations and graph theoretic terminology, we follow West [17] while the terms related to number theory are used in the sense of Burton [2].

We begin with finite, undirected and simple graph $G = (V(G), E(G))$ of order n. A set $D \subseteq V(G)$ of vertices in a graph G is called a dominating set if each vertex in $V(G) - D$ is adjacent to at least one vertex of D. A dominating set D is a minimal dominating set if no proper subset D' of D is a dominating set of graph G. The domination number $\gamma(G)$ is the minimum cardinality of a minimal dominating set.

The following new concept is recently introduced and further explored by Vaidya and Vadhel [13, 14, 15, 16].

A dominating set $D \subseteq V(G)$ is said to be a congruent dominating set of G if

$$\sum_{v\in V(G)} d(v) \equiv 0(\text{mod } \sum_{v\in D} d(v)) \quad (1)$$

A congruent dominating set $D \subseteq V(G)$ is said to be a minimal congruent dominating set if no proper subset D' of D is congruent dominating set. The minimum cardinality of a minimal congruent dominating set of G is called the congruent domination number of G which is denoted by $\gamma_{cd}(G)$.

In the present paper we have investigated the congruent domination number of some graph obtained by means of some graph operation like Corona product, square graph of a graph, complement graph of a graph and extended double cover of a graph. The domination number of the Cartesian product of paths and cycles have been investigated in [5, 10, 11]. We have also investigated the exact value of congruent domination number for Cartesian product of cycles and paths.

The complement $\bar{G}$ of a graph G is the graph with vertex set $V(G)$ and two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G.

The square graph G^2 of a graph G with vertex set $V(G)$ is the graph obtained by joining every pair of vertices which are at distance two in G.

The corona $G \circ H$ of two graphs G and H (with order n and m respectively) is defined as a graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

The Cartesian product of two graphs $G(V_1, E_1)$ and $H(V_2, E_2)$, denoted by $G \square H$, is the graph with vertex set is $V_1 \times V_2$ and edge set $E(G \square H) = \{((g_1, h_1), (g_2, h_2)): g_1 = g_2 \text{ and } (h_1, h_2) \in E_2 \text{ or } h_1 = h_2 \text{ and } (g_1, g_2) \in E_1\}$.