Application of Graph Theory in Various Field of Applied Science & Engineering

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Abstract: In this paper, the research has been focus on application of graph theory in various field of Science & Engineering like Chemistry in Caterpillar Trees, Time Table Scheduling in Academics, Communication Network and Computer Science.

Key words: Bipartite graph, Caterpillar graph, Graph labeling, Complete graph, Semi graceful labeling, Golomb Ruler.

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1. Introduction

Graph labeling is becoming more interesting filed due to its broad range of applications. A vital role has played by labeled graphs in various fields of graph theory. Coding theory, missile guidance codes, design of good Radar type codes, astronomy, circuit design, X-ray crystallography, data base management are few names of such important fields. This chapter gives an overview of graph labeling as well as some information of important applications.

Here I would like to enhance the graph labeling applications in the field of computer science. Graph labeling applications have been studied and here I explore the usage of this field in several areas like communication networks, image processing, data mining, crypto systems and bird view has been proposed. Graph theory has been applied in investigation of electrical network is a collection of components and device interconnected electrical gazettes.

The network components are idealized physical devices and system, in order to represent several properties. Also they must obey the Kirchhoff's law of currents and voltage.

2. Bipartite Graph and Time Table Scheduling

Allocation of classes and subject to all the teachers in an institute is one of major issues, whenever constrain and complexity occur. A bipartite graph helps to solve such problem. Also it play an important role in this kind of problems. For m teachers and n subjects available periods p, the time table has to be prepared as follow. A bipartite graph G, I mean a set of teachers $v_1, v_2, v_{3,} . . . , v_m$ and another set of subjects $u_1, u_2, u_{3,} . . . , u_n$. These vertices have p_i periods. It is presumed that any one period, each teacher may engage almost one subject. Also each subject can be taught by maximum one teacher. For the first period, the time table for this single period correspond to a matching in the bipartite graph G and conversely, each matching correspond to a possible assignment of some teachers to subject taught during that period.

Hence, the solution for this will be obtained by partitioning the edges of the given graph into minimum number of matching. Also the edge has to be colored with minimum number of colors and this problem can be solved by the vertex coloring algorithm. The line graph of given graph has equal number of vertices and edges of the given graph. Also the vertices in the line graph are adjacent iff they are incident in the given graph. The line graph is a simple graph and its proper coloring gives a proper edges coloring of the given graph.

3. Application in Communication Network

For any kind of application, it depends on problem scenario a kind of graph is used for representing the problem. a suitable labeling is applied on that graph in order to solve the problem. Given a set of transmitters, each station is assigned a channel number (a positive integer) such that interference should be avoided. The smaller distance between two stations has stronger interference. Hence, the difference in channel assignment has to be greater.

Here each vertex represents a transmitter and any its pair connected to the neighboring transmitters. Radio labeling is used to get effective network. For this, I define some terminology as take $G = (V(G), E(G))$ a connected graph and $d(u, v)$ = distance between any two vertices of G.

The Maximum distance between any pair of vertices is the diameter of G and denote it by $diam(G)$. A radio labeling on G, I mean an injective function $f: V(G) \rightarrow N \cup \{0\}$ and define it such a way so that for any $u, v \in V(G), |f(u) - f(v)| \geq diam(G) - d(u, v) + 1$. The span of f is the difference of the largest and the smallest channel used, that is $max \{ f(u) f(v)\}$, for every $u, v \in$

$V(G)$.. The radio number of the given graph G is the maximum span of radio labeling of G and denote it by γ_n(G). Given a set of transmitters, each station is defined as a channel such that the interference can be minimise or avoided.

Radio labeling process proved as an efficient way of determining the time of communication for sensor network. The network is considered as chain graph in which every sensor planted in the given network. Also it is a communication at time t, where t is radio channel assignment. Channel labeling can be used to determine the time at which the sensor communicate.

Radio labeling on different kind of graphs are shown in F*igure* 1.1.

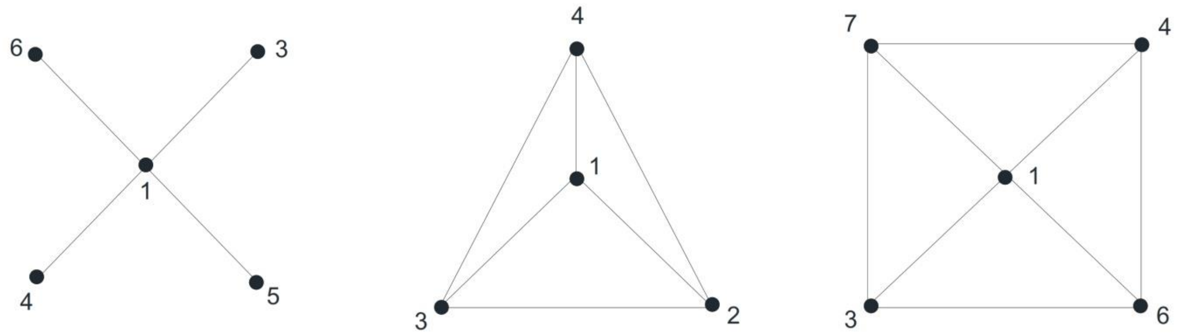

Figure 1.1

4. Caterpillar Trees in Chemistry

Combinatorial as well as physical properties of benzenoid hydrocarbons can be studied by study of related caterpillars. The simple way of defining a caterpillar by $S(x_{1,}\ x_{2,}\ .\ .\ .\ ,x_n)$. When all the pendent vertices of $S(x_1, x_{2,}\ .\ .\ .\ ,x_n)$ are deleted, it leaves a path P_n. Here x_i denotes number of pendant vertices, which are adjacent to i^{th} vertex of the path P_n obtained from $S(x_{1,}\ x_{2,}\ .\ .\ .\ ,x_n)$ by removing all the pendent vertices.

In chemistry, use of such trees are resulted from studying of the topological properties of benzenoid hydrocarbons are called resonant if a set of three circularly conjugated double bond can be drawn in both of them such that the rest of the carbon atoms are spanned either by a double bond or by a sextet of electrons. So, two edge in a caterpillar are incident iff the corresponding hexagons in the zenoid system are non resonant.

There is a one to one correspondence between the labeling of the edges of a caterpillar and those of the hexagons of a benzenoid system. Explicitly, these terms are considered in chemistry under the name of *Gutman Trees*. It is amazing that all graphs played an important role in chemical graph theory which is related to caterpillars. That is why such object have an important role for understanding and simplifying combinatorial properties of much more complicated graphs.

Potential of such trees in data reduction, computational graph theory and ordering of graph are considered. Thus, it becomes possible to study or properties of large graphs such as bezenoid graphs in terms of much smaller trees.

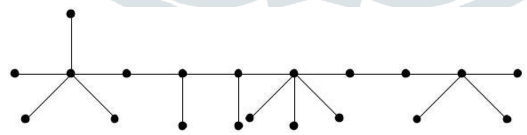

Figure 1.2

5. Application of Graph Labeling in Electrical Networks

Kirchhoff's circuit laws are two equalities, which deal with potential and current. They states the sum of all voltages around a loop is zero and the total resistance of n resistors in series is $RT = R_1 + R_2 +. . . +R_n$.

An electric circuit is complete and current flow from negative terminals of the power source. An electrical circuit is categories into three type namely series, parallel and both. The representation of graph in circuit network are one of the type of representation of graph and in the graph the current flows in circuit and present the linking of connection between resistors series and parallel connection are determined in the circuit.

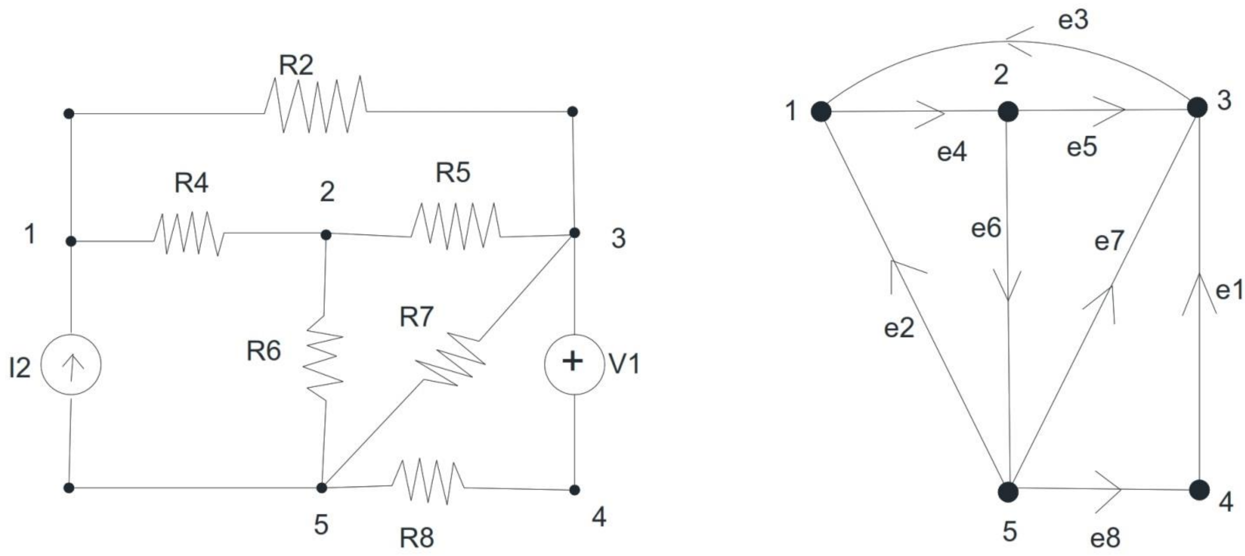

Figure 1.3

6. Application Computer Science

6.6.1: Communication Network

Conventional network represent global nature. Gigantic graphs are everywhere from communication network. Arithmetical representation of the graph structure inflict on these data sets. It provide to visualizing and understanding of study of the data. As I discussed in 1.1 that the radio labeling provides communication effectively. Graph labeling plays role in sensor network, adhoc network etc.

6.6.2: Data Mining

Graph mining represents the relational attribute of the data. There are five hypothetical based approaches in data mining. Subgraph categories, isomorphism, graph invariants, mining measures and many other mining measures that are very commonly used in machine learning filed are names of approaches of graph based data mining approach especially information entropy and information gain as well as solution method.

6.6.3: Web Designing

In a web graph, web pages are represented by vertices, the hyper links by edges and using it find one of the attractive information. Another application of graphs are website community, in which vertices are classes of the object and each vertex are adjacent each other. In graph theory, such graph is called *Complete graph* on n-vertices of K_n.

7. Golomb Rulers

It is obvious that K_n is graceful iff $n \leq 4$. By this fact Golomb motivated to define a new classes namely semi graceful labeling. According to him if the constraint edge labels to be consecutive integers is relaxed them such labeling is called **semi graceful labeling**. Semi graceful labeling is optional if it minimizes the largest edge label of K_n, which denoted by $G(K_n)$. Using it, Golomb got semi graceful labeling for K_5, which consist {1,2,3,4,5,6,7,8,9,10,11} edge labels And {0, 1, 4, 9, 11} vertex labels. Golomb has observed equivalence for the coding theory context between a semi graceful labeling, it helps to minimize $G(K_n)$. Using this observation he developed a special ruler on which n division marks are placed and it highlighted by Golomb Ruler. He produced vertices sets {0, 1, 4, 10, 12, 17} and {0, 1, 4, 10, 18, 23, 25} for K_6 and K_7 respectively to Minimize G (K_n), where n = 6, 7 respectively.

Following Table summarized the particular regarding possible semi graceful labeling for K_n (n = 10).

$\lvert V(K_n)\rvert$	$\lvert E(K_n)\rvert$	Division	Label of Vertices
2	1	1	0,1
3	3	1,2	0,1,3
4	6	1,3,2	0,1,4,6
5	11	1,3,5,2	0,1,4,9,11
		2,5,1,3	0,2,7,8,11
		1,3,6,2,5	0,1,4,10,12,17
6	17	1,3,6,5,2	0,1,4,10,15,17
		1,7,3,2,4	0,1,8,11,13,17
		1,7,4,2,3	0,1,8,12,14,17
7	25	1,3,6,8,5,2	0,1,4,10,18,23,25
		1,6,4,9,3,2	0,1,7,11,20,23,25
		1,10,5,3,4,2	1,1,11,16,29,23,25
		2,1,7,6,5,4	0,2,3,10,16,21,25
		2,5,6,8,1,3	0,2,7,13,21,22,25
8	34	1,3,5,6,7,10,2	0,1,4,9,15,22,32,34
9	44	1,4,7,13,2,8,6,3	0,1,5,12,25,27,35,41,44
10	55	1,5,4,13,3,8,7,12,2	0,1,6,10,23,26,34,41,53,55

Semigraceful labeling and Golomb ruler are shown in Figure 1.4.

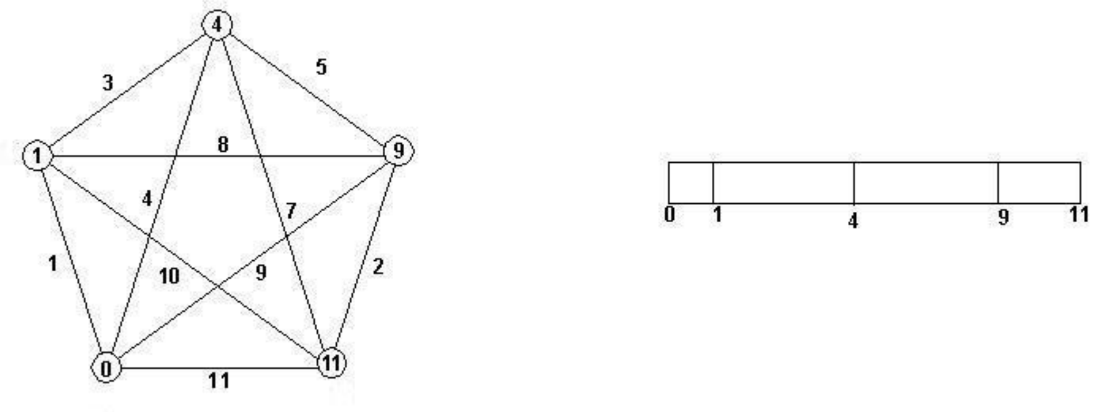

Figure 1.4

Similarly if I take the ruler R with 6 marks placed at 0,1, 4,10,12 and 17 semigraceful labeling Golomb ruler is shown in Figure 1.5.

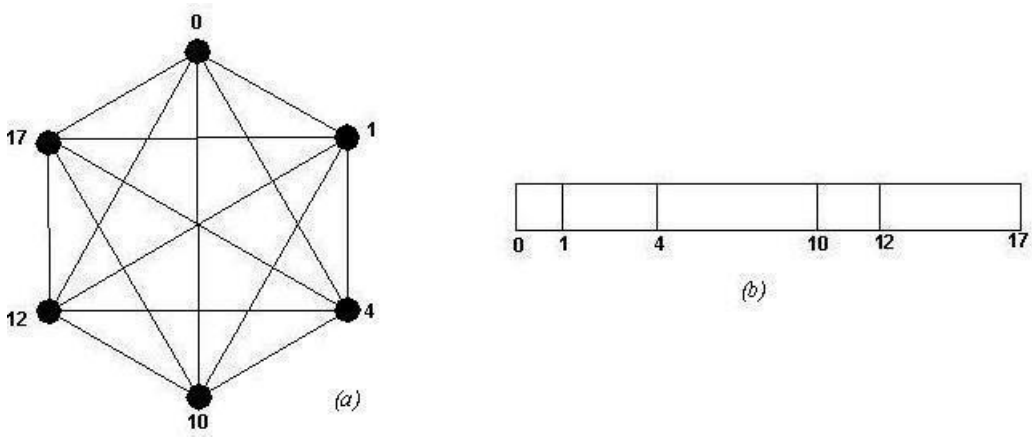

Figure 1.5

8. Conclusive Remarks

For the scope of future research, one can explore the related ruler problems which have similar applications in communication network. Such field includes the problem of finding the shortest ruler with k marks which measure all integer lengths form 1 to n. Also one can study the structure of different crystals using the ruler model. This approach would be helpful in interdisciplinary research topics.

Graph labeling present a common circumstances for many theoretical and applied problems. In this paper i have tried to explore some graph labeling applications and bird view of some graph theoretical applications which does not includes graph labeling techniques. Researchers may get some information related to graph labeling and its applications in the field of computer science. I also focused on the application of graphs to electrical network. These topics create an impression of graph labeling as a unifying model. It has vital potential to provide partial or complete solution for practical problems. This techniques may work as a powerful unifying model with some filed like bio-technology, information technology and new generation communication network

9. References

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