Chapter 4
Singular Value Decomposition-SVD based Watermarking

4.1 Introduction

SVD is a mathematical technique rooted in linear algebra and commonly used for matrix diagonalization in numerical computations. SVD has extensive applications in fields such as signal processing, image analysis, and data compression. When applied to an image matrix A of size M×N, it decomposes the matrix into three distinct components: U, S, and V, each possessing unique properties that make SVD highly effective for tasks like digital watermarking.

4.1.1 Properties of SVD

1. Matrix Decomposition: The image matrix A can be expressed as: $A = USV^T$. Here, U and V are orthogonal matrices, while S is a diagonal matrix.
2. Characteristics of the Decomposed Matrices:
    - U: Known as the left singular matrix, it is of size M×M.
    - V: Known as the right singular matrix, it is of size N×N.
    - S: A diagonal matrix of size M×N, with its elements (singular values) arranged in descending order along the diagonal.
3. Importance in Watermarking:
    - The singular matrix S is of particular interest in watermarking applications due to its stability. Even when minor alterations are made to the original image (cover medium), the singular values in S exhibit negligible changes.
    - Singular values encode critical information about the brightness of an image, while the singular vectors describe its geometric structure.
    - The stability of singular values ensures that watermarking operations do not significantly distort the original image.
4. Reconstruction Properties:
    - The singular values in S gradually decrease in magnitude. Smaller singular values contribute less to the overall image structure. As a result, even if some of these smaller values are omitted, the reconstructed image remains visually similar to the original.

    - This property allows for embedding watermark information without significantly affecting the image quality.
5. Efficient Representation:
    - SVD provides an algebraic representation of the fundamental features of an image.
    - Brightness characteristics are derived from singular values, while geometric attributes are captured by the singular vectors.

4.1.2 Practical Example of SVD

To illustrate the SVD process, consider an example where a matrix A undergoes decomposition into U, S, and V. Table 4.1 presents the decomposition, demonstrating how these components interact.

Original Matrix		
1	2	3
4	5	6
7	8	9
10	11	12

U Matrix			
0.140877	-0.82471	-0.54556	0.048576
0.343946	-0.42626	0.691212	-0.47141
0.547016	-0.02781	0.254268	0.797087
0.750086	0.370637	-0.39992	-0.37426

V Matrix		
0.504533	0.760776	0.408248
0.574516	0.057141	-0.8165
0.644498	-0.64649	0.408248

S Matrix		
25.46241	0	0
0	1.290662	0
0	0	1.46E-15
0	0	0

Table 4.1: Example of SVD

4.2 Advantages of SVD in Digital Watermarking

- **Versatility:**
  SVD supports the embedding of both binary and grayscale watermarks. This capability makes it more advanced compared to earlier methods that lacked such flexibility.
- **Robustness:**
  Due to the stability of singular values, watermarked images are resilient to attacks such as compression, noise addition, or cropping.
- **Minimal Distortion:**
  By modifying only the singular values, watermark embedding introduces minimal perceptual changes to the host image, maintaining its visual fidelity.

In conclusion, SVD stands out as a robust and flexible technique for digital watermarking. Its ability to embed different types of watermarks, combined with its stability and minimal impact on the host image, makes it an invaluable tool in modern multimedia security applications.

4.3 Applications of SVD in Watermarking

1. **Image Watermarking:**
   SVD is widely used to embed watermarks in digital images. The robustness of singular values ensures that the watermark remains intact even when the host image undergoes transformations such as compression, scaling, or rotation.
2. **Video Watermarking:**
   In addition to images, SVD enables the embedding of watermarks in video frames. This is achieved by applying SVD to individual frames, ensuring that the watermark persists across multiple frames while remaining imperceptible.
3. **Audio Watermarking:**
   SVD can also be applied in the audio domain by decomposing audio signals into singular values and vectors. This approach is particularly effective for embedding imperceptible watermarks in music and other audio recordings.
4. **Hybrid Techniques:**
   SVD is often combined with other transform techniques like Discrete Cosine Transform (DCT) or Discrete Wavelet Transform (DWT) to enhance the robustness and capacity of the watermarking process. The hybrid approach leverages the strengths of each method to improve resistance against attacks and ensure higher watermark fidelity.

4.4 Advantages of SVD-Based Watermarking

1. **Robustness:**
   - SVD-based watermarking is highly resilient to common image processing attacks like compression, noise addition, and geometric transformations.
2. **Minimal Distortion:**
   - By modifying only the singular values, the visual quality of the host image remains nearly unchanged.
3. **Flexibility:**
   - The technique supports a wide range of watermark types, including binary, grayscale, and even color watermarks.
4. **Stability of Singular Values:**

   - Singular values remain largely unaffected by minor alterations in the image, ensuring that the watermark is not easily compromised.

4.5 Limitations of SVD-Based Watermarking

1. **Computational Complexity:**
   - The decomposition process can be computationally intensive, especially for large images or videos.
2. **Limited Capacity:**
   - The amount of watermark data that can be embedded is limited by the size and stability of the singular values.
3. **Dependency on the Host Medium:**
   - Accurate extraction of the watermark requires access to the original host image or its singular values, which may not always be practical.

4.6 Embedding Process

The watermarking process using SVD generally involves the following steps:

1. **Decomposition:**
   - The host image is decomposed using SVD to obtain matrices U, S, and V.
   - The singular values in S are modified to embed the watermark.
2. **Watermark Embedding:**
   - The watermark is encoded into the singular values of the host image.
   - This embedding process ensures that the changes introduced by the watermark are distributed across the image, minimizing visible distortions.
3. **Reconstruction:**
   - The modified singular values are combined with the original U and V matrices to reconstruct the watermarked image.
   - The inverse SVD is applied to generate the final watermarked image.

4.7 Extraction Process

1. Decomposition of the Watermarked Image:
   - The watermarked image is decomposed into its U, S, and V matrices using SVD.
2. Watermark Recovery:

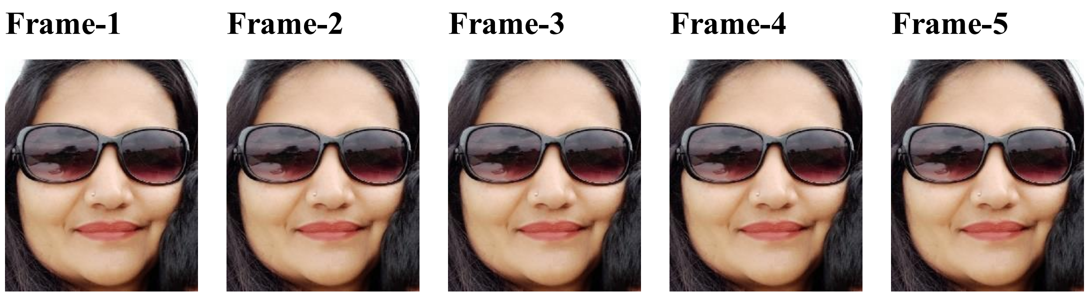
- The difference between the singular values of the watermarked image and the original image is used to extract the watermark.
- The recovered watermark is further processed to ensure that it is accurately reconstructed.

4.8 SVD-Based Watermarking with Binary Messages

4.8.1 Embedding Process

The binary watermark embedding process involves the following steps:

1. Split the original video into individual frames.
2. Faces are identified and located within the frame using algorithms such as Viola-Jones or deep learning-based detectors.
3. Convert each frame from the RGB color space to YCbCr.
4. Use the Y component for embedding the watermark.
5. Apply SVD to the Y frame to obtain its singular values.
6. Resize the watermark to match the dimensions of the singular component S.
7. Modify the singular values using S=S+K·W, where K is the gain factor, and W is the watermark.
8. Perform SVD on the modified singular values.
9. Reconstruct the modified sub-band using U·Smodified·VT.
10. The watermarked face image, now containing the embedded watermark, is reinserted back into its original position within the image
11. Convert the frame back to RGB and repeat the process for all frames.
12. Combine all watermarked frames to create the final watermarked video.



(a)

Watermarked: 1	Watermarked: 2	Watermarked: 3	Watermarked: 4	Watermarked: 5

(b)

Figure 4.1: SVD with K=100 (a) 5 Frames of video (b) Watermarked Frames

4.8.2Extraction Process

To retrieve the embedded watermark, the following steps are used:

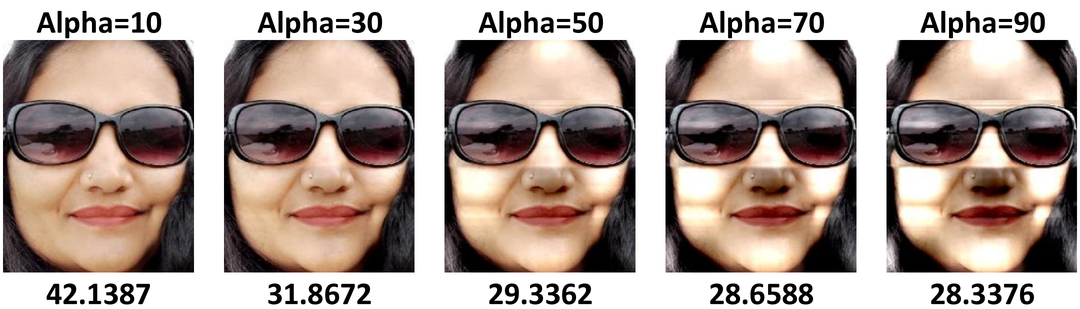
1. Split the watermarked video into frames.
2. Faces are identified and located within the frame using algorithms such as Viola-Jones or deep learning-based detectors.
3. Convert frames to YCbCr, selecting the Y component.
4. Apply SVD to the Y frame to extract its singular values.
5. Compute the watermark using W=(D−S)/K, where D is the modified singular matrix.
6. Repeat the process for all frames to recover the watermark.

Figure 4.2 shows the extracted message with gain factor 100.

Recovered:1	Recovered:2	Recovered:3	Recovered:4	Recovered:5

Figure 4.2: Recovered Messages

Results



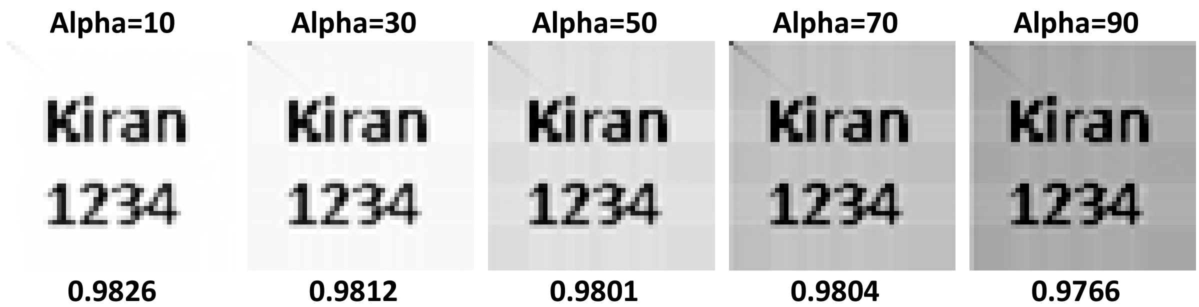


Figure 4.3: Various values of K (a) Frame 1-Watermarked (b) Messages at receiver end

Frame No.	PSNR (db)	MSE	Correlation
1	28.2331	97.5656	0.9634
2	28.5446	98.4876	0.9668
3	28.9832	97.4147	0.9694
4	29.2302	97.7542	0.9601
5	28.9876	96.9542	0.9687

Table 4.2 SVD with Various Frames with K=100

Alpha	PSNR (db)	MSE	Correlation
10	42.138	3.9739	0.9826
20	43.4634	18.3883	0.982
30	31.8672	42.3019	0.9812
40	30.0935	63.6407	0.9805
50	29.3362	75.7643	0.9801
60	28.9253	83.2827	0.9798
70	28.6588	88.5521	0.9804
80	28.4789	92.2977	0.9777
90	28.3376	95.3506	0.9766
100	28.2446	97.4147	0.9694

Table 4.3 SVD with various values of K.

4.9 Results and Observations

- Increasing the gain factor enhances robustness but reduces perceptibility.
- Frames with PSNR values above 28 dB are visually acceptable, while correlation values above 0.50 ensure message recognition.
- The method demonstrates resilience against various attacks, including compression, filtering, and noise addition.

4.10 Comparative Observations

1. Perceptibility:
    - SVD-based methods maintain higher perceptibility compared to correlation-based approaches.
    - Slightly less perceptibility compared to DCT and DWT methods.
2. Robustness:
    - SVD-based watermarking exhibits maximum robustness at the same gain factor compared to DCT, DWT, and correlation methods.