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On total domination and total equitable domination in graphs

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Abstract
A dominating set D of a graph G is called total if every vertex of $V(G)$ is adjacent to at least one vertex of D, equivalently if $N(D) = V(G)$ then D is called total dominating set. A dominating set D is called total equitable dominating set if it is total and for every vertex in $V(G) - D$ there exists a vertex in D such that they are adjacent and difference between their degrees is at most one. The minimum cardinality of a total (total equitable) dominating set is called total (total equitable) domination number of G which is denoted by $\gamma_t(G)(\gamma_t^e(G))$. We have investigated exact value of these parameters for some graphs.

Keywords
Dominating set, total dominating set, equitable dominating set.

AMS Subject Classification
05C69; 05C76.

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Contents

1. Introduction

Theory of domination has been remained in the focus of many researchers because of its applications in variety of fields such as linear algebra, optimization, social science, design of communication networks and military surveillance. Many variants of domination models such as total domination [3], equitable domination [10], fractional domination [4], global domination [9] are among mention a few. Some more variants of domination models are also explored in [1, 7, 8, 14]. This paper is focused on total domination and total equitable domination in graphs.

We begin with simple, finite, undirected and connected graph $G = (V(G), E(G))$. For a vertex $v \in V(G)$, the open neighborhood $N(v)$ is defined as $N(v) = \{u \in V(G) : uv \in E(G)\}$. The maximum degree among the vertices of G is denoted by $\Delta(G)$. For any real number n, $\lceil n \rceil$ denotes the smallest integer not less than n while $\lfloor n \rfloor$ denotes the greatest integer not greater than n.

Definition 1.1. *The wheel W_n with n vertices is defined to be the join of K_1 and C_{n-1}. The vertex corresponding to K_1 is known as apex while the vertices corresponding to C_n are known as rim vertices.*

Definition 1.2. *The switching of a vertex v of G means removing all the edges incident to v and adding edges joining v to every vertex which is not adjacent to v in G. We denote the resultant graph by $\widetilde{G}$.*

The set $D \subseteq V(G)$ of vertices in a graph G is called dominating set if every vertex $v \in V(G)$ is either an element of D or is adjacent to an element of D. The minimum cardinality of a dominating set of G is called the domination number of G which is denoted by $\gamma(G)$.

A subset D of $V(G)$ is called a total dominating set of G if $N(D) = V(G)$ or equivalently if every vertex $v \in V(G)$ is adjacent to at least one element in D. The minimum cardinality of total dominating set is called total domination number which is denoted by $\gamma_t(G)$. This concept was introduced by Cockayne *et al.* [3]. A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G) - D$ there exists a vertex

$u \in D$ such that $uv \in E(G)$ and $|d(v) - d(u)| \leqslant 1$, where $d(u)$ denotes the degree of vertex u and $d(v)$ denotes the degree of vertex v. The minimum cardinality of such dominating set is called the equitable domination number of G which is denoted by $\gamma^e(G)$. This concept was introduced by Swaminathan and Dharmalingam [10]. A dominating set which is both total and equitable is called total equitable dominating set. The total equitable domination number of G is the minimum cardinality of a total equitable dominating set of G which is denoted by $\gamma_t^e(G)$. This concept was introduced by Basavanagoud *et al.* [2] and further explored by Vaidya and Parmar [11, 12]. The concept of total equitable bondage number possesses the blend of total equitable domination as well as bondage number in graphs. This concept was introduced by Vaidya and Parmar [13].

For standard terminology and notation in graph theory we rely upon West [15] while for any undefined term related to theory of domination we refer to Haynes *et al.* [5].

We state some existing results for our ready reference.

Proposition 1.3. *[3] If G is a connected and $\Delta(G) \leq n-2$ then $\gamma_t(G) \leq n - \Delta(G)$.*

Proposition 1.4. *[6] If G is a graph of order n with no isolated vertex then $\gamma_t(G) \geq \dfrac{n}{\Delta(G)}$.*

Proposition 1.5. *[2] For any nontrivial connected graph G, $\left\lceil \dfrac{n}{\Delta(G)} \right\rceil + 1 \leq \gamma_t^e(G)$.*

2. Main Results

Theorem 2.1. *If $\widetilde{P_n}$ is the graph obtained by switching of a pendant vertex of path P_n $(n > 3)$ then $\gamma_t(\widetilde{P_n}) = 2$.*

Proof: Let $\widetilde{P_n}$ be the graph obtained by switching of pendant vertex v_1 of P_n. Let $V(\widetilde{P_n}) = V(P_n) = \{v_1, v_2, ..., v_n\}$ be the vertex set where $d_{\widetilde{P_n}}(v_1) = n-2 = \Delta(\widetilde{P_n})$, $d_{\widetilde{P_n}}(v_2) = 1$, $d_{\widetilde{P_n}}(v_n) = 2$ and $d_{\widetilde{P_n}}(v_i) = 3$, $\forall i \in \{1, 2, 3, ..., n-1\}$.

Now $\widetilde{P_n}$ is the graph with no isolated vertices. Hence $\gamma_t(\widetilde{P_n}) \geq 2$ by Proposition 1.4. Also $\gamma_t(\widetilde{P_n}) \leq 2$ by Proposition 1.3, as the graph $\widetilde{P_n}$ is connected and $\Delta(\widetilde{P_n}) \leq n-2$. Hence $\gamma_t(\widetilde{P_n}) = 2$.

Theorem 2.2. *If $\widetilde{P_n}$ is the graph obtained by switching of a pendant vertex of path P_n $(n > 4)$ then*

$$\gamma_t^e\left(\widetilde{P_n}\right) = \begin{cases} 2 & ; \text{if } n = 3 \\ 3 & ; \text{if } n = 5 \\ \left\lceil \dfrac{n}{3} \right\rceil + 2 & ; \text{if } n \equiv 2(\bmod 3) \\ \dfrac{n}{3} + 2 & ; \text{if } n \equiv 0(\bmod 3) \\ \left\lfloor \dfrac{n}{3} \right\rfloor + 2 & ; \text{if } n \equiv 1(\bmod 3) \end{cases}$$

Proof: Let $\widetilde{P_n}$ be the graph obtained by switching of pendant vertex v_1 of P_n. Let $V(\widetilde{P_n}) = V(P_n) = \{v_1, v_2, ..., v_n\}$ be the vertex set where $d_{\widetilde{P_n}}(v_1) = n-2$, $d_{\widetilde{P_n}}(v_2) = 1$, $d_{\widetilde{P_n}}(v_n) = 2$ and $d_{\widetilde{P_n}}(v_i) = 3$, $\forall i \in \{1, 2, 3, ..., n-1\}$.

Suppose D is any total equitable dominating set of $\widetilde{P_n}$ then it is obvious that v_1 must belong to D as $d(v_1) = n-2 = \Delta(\widetilde{P_n})$.

To prove the result we consider the following cases:

Case 1: For $n = 3$

In this case $\widetilde{P_3}$ is isomorphic to P_3. Hence $\gamma_t^e(\widetilde{P_3}) = 2$.

Case 2: For $n = 5$

By Proposition 1.5, $\gamma_t^e(\widetilde{P_5}) \geq 3$. Hence $\gamma_t^e(\widetilde{P_5}) = 3$.

Case 3: For $n \equiv 2(\bmod 3), n \neq 5$

We construct a set of vertices D is as follows:

$$D = \left\{ v_{3i} : 1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1 \right\} \cup \{v_1, v_2, v_n\}$$

Then $|D| = \left\lceil \frac{n}{3} \right\rceil + 2$. Also every vertex $v \in V(\widetilde{P_n}) - D$ is adjacent to at least two vertices of D and moreover $d_{\widetilde{P_n}}(v) = 3$. Thus D is an equitable dominating set of $\widetilde{P_n}$. Further D is a total dominating set of $\widetilde{P_n}$ as $N(D) = V(\widetilde{P_n})$. Therefore D is a total equitable dominating set of $\widetilde{P_n}$.

For any $v \in D$ (where $v \neq v_1$), the set $D - \{v\}$ is not equitable dominating set of $\widetilde{P_n}$ as for all vertex $u \in V(\widetilde{P_n}) - (D - \{v\})$ there does not exists $w \in D - \{v\}$ such that $|d_{\widetilde{P_n}}(u) - d_{\widetilde{P_n}}(w)| \leq 1$. If $v = v_1$ then $N(D - \{v_1\}) \neq V(\widetilde{P_n})$ therefore $D - \{v_1\}$ is not total dominating set of $\widetilde{P_n}$. Therefore D is a minimal total equitable dominating set of $\widetilde{P_n}$.

Hence, $\gamma_t^e(\widetilde{P_n}) = \left\lceil \frac{n}{3} \right\rceil + 2$, when $n \equiv 2(\bmod 3)$.

Case 4: For $n \equiv 0(\bmod 3)$

We construct a set of vertices D is as follows:

$$D=\left\{v_{3i}:1\le i\le \frac{n}{3}\right\}\cup\{v_1,v_2\}$$

Then $|D|=\frac{n}{3}+3$. Also every vertex $v\in V(\widetilde{P_n})-D$ is adjacent to at least two vertices of D and also $d_{\widetilde{P_n}}(v)=3$. Thus D is an equitable dominating set of $\widetilde{P_n}$. Further D is a total dominating set of $\widetilde{P_n}$ as $N(D)=V(\widetilde{P_n})$. Therefore D is a total equitable dominating set of $\widetilde{P_n}$.

For any $v\in D$ (where $v\neq v_1$), the set $D-\{v\}$ is not equitable dominating set of $\widetilde{P_n}$ as for all vertex $u\in V(\widetilde{P_n})-(D-\{v\})$ there does no exists $w\in D-\{v\}$ such that $|d_{\widetilde{P_n}}(u)-d_{\widetilde{P_n}}(w)|\le 1$. If $v=v_1$ then $N(D-\{v_1\})\neq V(\widetilde{P_n})$ therefore $D-\{v_1\}$ is not total dominating set of $\widetilde{P_n}$. Therefore D is a minimal total equitable dominating set of $\widetilde{P_n}$.

Hence, $\gamma_t^e(\widetilde{P_n})=\frac{n}{3}+2$, when $n\equiv 0(\bmod 3)$.

Case 5: For $n\equiv 1(\bmod 3)$

We construct a set of vertices D is as follows:

$$D=\left\{v_{3i}:1\le i\le \left\lfloor\frac{n}{3}\right\rfloor\right\}\cup\{v_1,v_2\}$$

Then $|D|=\left\lfloor\frac{n}{3}\right\rfloor+2$. Also every vertex $v\in V(\widetilde{P_n})-D$ is adjacent to at least two vertices of D and moreover $d_{\widetilde{P_n}}(v)=2$ or 3. Thus D is an equitable dominating set of $\widetilde{P_n}$. Further D is a total dominating set of $\widetilde{P_n}$ as $N(D)=V(\widetilde{P_n})$. Therefore D is a total equitable dominating set of $\widetilde{P_n}$.

For any $v\in D$ (where $v\neq v_1$), the set $D-\{v\}$ is not equitable dominating set of $\widetilde{P_n}$ as for all vertex $u\in V(\widetilde{P_n})-(D-\{v\})$ there does not exists $w\in D-\{v\}$ such that $|d_{\widetilde{P_n}}(u)-d_{\widetilde{P_n}}(w)|\le 1$. If $v=v_1$ then $N(D-\{v_1\})\neq V(\widetilde{P_n})$ therefore $D-\{v_1\}$ is not total dominating set of $\widetilde{P_n}$. Therefore D is a minimal total equitable dominating set of $\widetilde{P_n}$.

Hence, $\gamma_t^e(\widetilde{P_n})=\left\lfloor\frac{n}{3}\right\rfloor+2$, when $n\equiv 1(\bmod 3)$.

Theorem 2.3. *If $\widetilde{C_n}$ is the graph obtained by switching of an arbitrary vertex of cycle C_n ($n>3$) then*

$$\gamma_t\left(\widetilde{C_n}\right)=\begin{cases}2 & ;\text{ if } n=4,\ 5\\ 3 & ;\text{ if } n>6\end{cases}$$

Proof: Let $\widetilde{C_n}$ be the graph obtained by switching of an arbitrary vertex(say v_1) of C_n. Let $V(\widetilde{C_n})=\{v_1,v_2,...,v_n\}$ be the vertex set of $\widetilde{C_n}$ with $d_{\widetilde{C_n}}(v_1)=n-3=\Delta(\widetilde{C_n})$, $d_{\widetilde{C_n}}(v_2)=d_{\widetilde{C_n}}(v_n)=1$ and $d_{\widetilde{C_n}}(v_i)=3;\forall i\in\{3,4,...,n-1\}$.

Suppose D is any total dominating set of $\widetilde{C_n}$. As the graph $\widetilde{C_n}$ has no isolated vertices then by Proposition 1.3, $\gamma_t(\widetilde{C_n})\ge 2$. Also the graph $\widetilde{C_n}$ is connected and $\Delta(\widetilde{C_n})\le n-2$ then by Proposition, 1.4, $\gamma_t(\widetilde{C_n})\le 2$. Thus $2\le\gamma_t(\widetilde{C_n})\le 3$.

To prove the result we consider the following cases:

Case 1: For $n=4,5$

For $n=4$, $\gamma_t(\widetilde{C_4})=2$ as $\widetilde{C_4}$ is isomorphic to $K_{1,3}$.

For $n=5$, we construct a set of vertices $D=\{v_3,v_4\}$. Then $|D|=2$. Thus D is of minimum cardinality $\gamma_t(\widetilde{C_n})\ge 2$. Therefore $\gamma_t(\widetilde{C_5})=2$.

Hence, $\gamma_t(\widetilde{C_n})=2$, when $n=4,5$.

Case 2: For $n>6$

For $n>6$, we construct a set of vertices $D=\{v_1,v_3,v_{n-1}\}$ with $|D|=3$. Moreover D is a total dominating set of $\widetilde{C_n}$ as $N(D)=V(\widetilde{C_n})$.

Further D is a minimum total dominating set of $\widetilde{C_n}$ because for any $v\in D$, $N(D-\{v\})\neq V(\widetilde{C_n})$.

Hence, $\gamma_t(\widetilde{C_n})=3$, when $n>6$.

Theorem 2.4. *If $\widetilde{C_n}$ is the graph obtained by switching of an arbitrary vertex of cycle C_n ($n>3$) then*

$$\gamma_t^e\left(\widetilde{C_n}\right)=\begin{cases}4 & ;\text{ if } n=4,\ 5,\ 6\\ \left\lceil\frac{n}{3}\right\rceil+3 & ;\text{ if } n\equiv 2(\bmod 3)\\ \frac{n}{3}+3 & ;\text{ if } n\equiv 0(\bmod 3)\\ \left\lfloor\frac{n}{3}\right\rfloor+3 & ;\text{ if } n\equiv 1(\bmod 3)\end{cases}$$

Proof: Let $\widetilde{C_n}$ be the graph obtained by switching of an arbitrary vertex(say v_1) of C_n. Let $V(\widetilde{C_n})=\{v_1,v_2,...,v_n\}$ be the vertex set of $\widetilde{C_n}$ with $d_{\widetilde{C_n}}(v_1)=n-3=\Delta(\widetilde{C_n})$, $d_{\widetilde{C_n}}(v_2)=d_{\widetilde{C_n}}(v_n)=1$ and $d_{\widetilde{C_n}}(v_i)=3;\forall i\in\{3,4,...,n-1\}$.

Suppose D is any total equitable dominating set of $\widetilde{C_n}$ then it is obvious that v_1 must belong to D as $d(v_1)=n-3=\Delta(\widetilde{C_n})$.

To prove the result we consider the following cases:

Case 1: For $n=4,5,6$

For $n = 4$, $\gamma_t^e(\widetilde{C_4}) = 4$ as $\widetilde{C_4}$ is isomorphic to $K_{1,3}$.

For $n = 5, 6$, we construct a set of vertices $D = \{v_2, v_3, v_{n-1}, v_n\}$ with $|D| = 4$. Also D is a total equitable dominating set $\widetilde{C_n}$ as every vertex $v \in V(\widetilde{C_n}) - D$ adjacent to two vertices of degree 3 from D. Thus D is equitable dominating set of $\widetilde{C_n}$ as $N(D) = V(\widetilde{C_n})$.

Further D is a minimal total equitable dominating set of $\widetilde{C_n}$ because for any $v \in D$, $N(D - \{v\}) \neq V(\widetilde{C_n})$.

Hence, $\gamma_t^e(\widetilde{C_n}) = 4$, when $n = 4, 5, 6$.

Case 2: For $n \equiv 2(\bmod 3)$ $n \neq 5$

We construct a set D of vertices is as follows:

$$D = \left\{v_{3i} : 3 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1\right\} \cup \{v_1, v_2, v_3, v_{n-1}, v_n\}$$

Then $|D| = \left\lceil \frac{n}{3} \right\rceil + 3$. Also every vertex $v \in V(\widetilde{C_n}) - D$ is adjacent to at least two vertices of D and moreover $d_{\widetilde{C_n}}(v) = 3$. Thus D is an equitable dominating set of $\widetilde{C_n}$. Further D is a total dominating set of $\widetilde{C_n}$ as $N(D) = V(\widetilde{C_n})$. Therefore D is a total equitable dominating set of $\widetilde{C_n}$.

For any $v \in D$, the set $D - \{v\}$ is not equitable dominating set of $\widetilde{C_n}$ as for all vertex $u \in V(\widetilde{C_n}) - (D - \{v\})$ there does not exists $w \in D - \{v\}$ such that $|d_{\widetilde{C_n}}(u) - d_{\widetilde{C_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{C_n}$.

Hence, $\gamma_t^e(\widetilde{C_n}) = \left\lceil \frac{n}{3} \right\rceil + 3$, when $n \equiv 2(\bmod 3)$, $n \neq 5$.

Case 3: For $n \equiv 0(\bmod 3)$

We construct a set D of vertices is as follows:

$$D = \left\{v_{3i} : 3 \leq i \leq \frac{n}{3} - 1\right\} \cup \{v_1, v_2, v_3, v_{n-1}, v_n\}$$

Then $|D| = \frac{n}{3} + 3$. Also every vertex $v \in V(\widetilde{C_n}) - D$ is adjacent to at least two vertices of D and moreover $d_{\widetilde{C_n}}(v) = 3$. Thus D is an equitable dominating set of $\widetilde{C_n}$. Further D is a total dominating set of $\widetilde{C_n}$ as $N(D) = V(\widetilde{C_n})$. Therefore D is a total equitable dominating set of $\widetilde{C_n}$.

For any $v \in D$, the set $D - \{v\}$ is not equitable dominating set of $\widetilde{C_n}$ as for all vertex $u \in V(\widetilde{C_n}) - (D - \{v\})$ there does not exists $w \in D - \{v\}$ such that $|d_{\widetilde{C_n}}(u) - d_{\widetilde{C_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{C_n}$.

Hence, $\gamma_t^e(\widetilde{C_n}) = \frac{n}{3} + 3$, when $n \equiv 0(\bmod 3)$.

Case 4: For $n \equiv 1(\bmod 3)$

We construct a set D of vertices is as follows:

$$D = \left\{v_{3i} : 3 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor\right\} \cup \{v_1, v_2, v_3, v_n\}$$

Then $|D| = \leq \left\lfloor \frac{n}{3} \right\rfloor + 3$. Also every vertex $v \in V(\widetilde{C_n}) - D$ is adjacent to at least two vertices of D and moreover $d_{\widetilde{C_n}}(v) = 3$. Thus D is an equitable dominating set of $\widetilde{C_n}$. Further D is a total dominating set of $\widetilde{C_n}$ as $N(D) = V(\widetilde{C_n})$. Therefore D is a total equitable dominating set of $\widetilde{C_n}$.

For any $v \in D$, the set $D - \{v\}$ is not equitable dominating set of $\widetilde{C_n}$ as for all vertex $u \in V(\widetilde{C_n}) - (D - \{v\})$ there exists $w \in D - \{v\}$ such that $|d_{\widetilde{C_n}}(u) - d_{\widetilde{C_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{C_n}$.

Hence, $\gamma_t^e(\widetilde{C_n}) = \left\lfloor \frac{n}{3} \right\rfloor + 3$, when $n \equiv 1(\bmod 3)$.

Theorem 2.5. *If $\widetilde{W_n}$ is the graph obtained by switching of a rim vertex of wheel W_n ($n > 5$) then $\gamma_t(\widetilde{W_n}) = 2$.*

Proof: Let $\widetilde{W_n}$ be the graph obtained by switching of an arbitrary vertex(say v_1) of W_n. Let $V(\widetilde{W_n}) = \{u, v_1, v_2, \ldots, v_{n-1}\}$ be the vertex set where $d_{\widetilde{W_n}}(u) = n - 2 = \Delta(\widetilde{W_n})$, $d_{\widetilde{W_n}}(v_1) = n - 4$, $d_{\widetilde{W_n}}(v_2) = d_{\widetilde{W_n}}(v_{n-1}) = 2$ and $d_{\widetilde{W_n}}(v_i) = 4$; $\forall i \in \{3, 4, \ldots, n-2\}$.

If D is any total dominating set of $\widetilde{W_n}$ then it is obvious that u must belong to D as $d_{\widetilde{W_n}}(u) = n - 2 = \Delta(\widetilde{W_n})$.

To prove this result we consider the following cases:

Case 1: For $n = 5$

We construct a set of vertices $D = \{v_1, v_3\}$ with $|D| = 2$. Also D is a total dominating set $\widetilde{W_5}$ and by Proposition 1.3, D is a minimal total dominating set of $\widetilde{W_5}$. Hence $\gamma_t(\widetilde{W_5}) = 2$.

Case 2: For $n > 5$

We construct a set of vertices $D\{u, v_i\}$ for any $i \in \{3, 4, \ldots, n-1\}$ with $|D| = 2$. Also D is a total dominating set of $\widetilde{W_n}$ and by Proposition 1.3, D is a minimal total dominating set of $\widetilde{W_n}$. Hence $\gamma_t(\widetilde{W_n}) = 2$, for $n > 5$.

Theorem 2.6. *If $\widetilde{W_n}$ is the graph obtained by switching of a* *rim vertex of wheel W_n ($n > 4$) then*

$$\gamma_t^e\left(\widetilde{W_n}\right) = \begin{cases} 3 & ; \text{if } n = 5 \\ 4 & ; \text{if } n = 7 \\ 5 & ; \text{if } n = 8,9 \\ \left\lceil \frac{n}{3} \right\rceil + 3 & ; \text{if } n \equiv 2(\bmod 3),\ n \neq 5,8 \\ \frac{n}{3} + 3 & ; \text{if } n \equiv 0(\bmod 3),\ n \neq 9 \\ \left\lfloor \frac{n}{3} \right\rfloor + 3 & ; \text{if } n \equiv 1(\bmod 3), n \neq 7 \end{cases}$$

Proof: Let $\widetilde{W_n}$ be the graph obtained by switching of an arbitrary vertex(say v_1) of W_n. Let $V(\widetilde{W_n}) = \{u, v_1, v_2, ..., v_{n-1}\}$ be the vertex set where $d_{\widetilde{W_n}}(u) = n-2 = \Delta(\widetilde{W_n})$, $d_{\widetilde{W_n}}(v_1) = n-4$, $d_{\widetilde{W_n}}(v_2) = d_{\widetilde{W_n}}(v_{n-1}) = 2$ and $d_{\widetilde{W_n}}(v_i) = 4$; $\forall i \in \{3, 4, ..., n-2\}$.

If D is any total dominating set of $\widetilde{W_n}$ then it is obvious that u must belong to D as $d_{\widetilde{W_n}}(u) = n-2 = \Delta(\widetilde{W_n})$.

To prove the result we consider the following cases:

Case 1: For $n = 5$

We construct a set of vertices $D = \{u, v_1, v_2\}$ with $|D| = 3$. Also every vertex $v \in V(\widetilde{W_5}) - D$ is adjacent to at least two vertices of D and there exists $u \in D$ such that $|d_{\widetilde{W_5}}(v) - d_{\widetilde{W_5}}(w)| \leq 1$. Thus D is an equitable dominating set of $\widetilde{W_5}$. Further D is a total dominating set of $\widetilde{W_5}$ as $N(D) = V(\widetilde{W_5})$. Therefore D is a total equitable dominating set of $\widetilde{W_5}$.

For any $v \in D$, the set $D - \{v\}$ is not equitable dominating set of $\widetilde{W_5}$ as for all vertex $u \in V(\widetilde{W_5}) - (D - \{v\})$ there does not exists $w \in D - \{v\}$ such that $|d_{\widetilde{W_5}}(u) - d_{\widetilde{W_5}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{W_5}$. Hence $\gamma_t^e(\widetilde{W_5}) = 3$.

Case 2: For $n = 7$

We construct a set of vertices $D = \{u, v_2, v_3, v_5\}$ with $|D| = 4$. Also every vertex $v \in V(\widetilde{W_7}) - D$ is adjacent to at least two vertices of D there exists $u \in D$ such that $|d_{\widetilde{W_7}}(v) - d_{\widetilde{W_7}}(w)| \leq 1$. Thus D is an equitable dominating set of $\widetilde{W_7}$. Further D is a total dominating set of $\widetilde{W_7}$ as $N(D) = V(\widetilde{W_7})$. Therefore D is a total equitable dominating set of $\widetilde{W_7}$.

For any $v \in D$, the set $D - \{v\}$ is not even an equitable dominating set of $\widetilde{W_7}$ as for all vertex $u \in V(\widetilde{W_7}) - (D - \{v\})$ there does not exist $w \in D - \{v\}$ such that $|d_{\widetilde{W_7}}(u) - d_{\widetilde{W_7}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{W_7}$. Hence $\gamma_t^e(\widetilde{W_7}) = 4$.

Case 3: For $n = 8, 9$

We construct a set of vertices $D = \{u, v_2, v_3, v_6, v_{n-1}\}$ with $|D| = 5$. Also every vertex $v \in V(\widetilde{W_n}) - D$ is adjacent to at least two vertices of D there exists $u \in D$ such that $|d_{\widetilde{W_n}}(v) - d_{\widetilde{W_n}}(w)| \leq 1$. Thus D is an equitable dominating set of $\widetilde{W_n}$. Further D is a total dominating set of $\widetilde{W_n}$ as $N(D) = V(\widetilde{W_n})$. Therefore D is a total equitable dominating set of $\widetilde{W_n}$.

For any $v \in D$, the set $D - \{v\}$ is not even an equitable dominating set of $\widetilde{W_n}$ as for all vertex $u \in V(\widetilde{W_n}) - (D - \{v\})$ there does not exist $w \in D - \{v\}$ such that $|d_{\widetilde{W_n}}(u) - d_{\widetilde{W_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{W_n}$. Hence $\gamma_t^e(\widetilde{W_n}) = 5$.

Case 4: For $n \equiv 2(\bmod 3)$, $n \neq 5, 8$

We construct a set D of vertices is follows:

$$D = \left\{v_{3i-1} : 2 \leq i \leq \left\lceil \frac{n}{3} \right\rceil - 1\right\} \cup \{u, v_1, v_2, v_3, v_4, v_{n-1}\}$$

with $|D| = \left\lceil \frac{n}{3} \right\rceil + 3$. Also for every vertex $v \in V(\widetilde{W_n}) - D$ is adjacent to at least two vertices of D, there exists $u \in D$ such that $|d_{\widetilde{W_n}}(v) - d_{\widetilde{W_n}}(w)| \leq 1$. Hence D is an equitable dominating set of $\widetilde{W_n}$. Further D is a total dominating set of $\widetilde{W_n}$ as $N(D) = V(\widetilde{W_n})$. Therefore D is a total equitable dominating set of $\widetilde{W_n}$.

For any $v \in D$, the set $D - \{v\}$ is not even an equitable dominating set of $\widetilde{W_n}$ as for all vertex $u \in V(\widetilde{W_n}) - (D - \{v\})$ there does not exist $w \in D - \{v\}$ such that $|d_{\widetilde{W_n}}(u) - d_{\widetilde{W_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{W_n}$.

Hence $\gamma_t^e(\widetilde{W_n}) = \left\lceil \frac{n}{3} \right\rceil + 3$, when $n \equiv 2(\bmod 3)$, $n \neq 5, 8$.

Case 5: For $n \equiv 0(\bmod 3)$, $n \neq 5$

We construct a set D of vertices is follows:

$$D = \left\{v_{3i} : 2 \leq i \leq \frac{n}{3} - 1\right\} \cup \{u, v_1, v_2, v_4, v_{n-1}\}$$

with $|D| = \frac{n}{3} + 3$. Also for every vertex $v \in V(\widetilde{W_n}) - D$ is adjacent to at least two vertices of D, there exists $u \in D$ such that $|d_{\widetilde{W_n}}(v) - d_{\widetilde{W_n}}(w)| \leq 1$. Hence D is an equitable dominating set of $\widetilde{W_n}$. Further D is a total dominating set of $\widetilde{W_n}$ as $N(D) = V(\widetilde{W_n})$. Therefore D is a total equitable dominating set of $\widetilde{W_n}$.

For any $v \in D$, the set $D - \{v\}$ is not even an equitable dominating set of $\widetilde{W_n}$ as for all vertex $u \in V(\widetilde{W_n}) - (D - \{v\})$ there does not exist $w \in D - \{v\}$ such that $|d_{\widetilde{W_n}}(u) - d_{\widetilde{W_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{W_n}$.

Hence $\gamma_t^e(\widetilde{W_n}) = \frac{n}{3} + 3$, when $n \equiv 0(\bmod 3)$, $n \neq 9$.

Case 6: For $n \equiv 1(\bmod 3)$

We construct a set D of vertices is follows:

$$D = \left\{ v_{3i+1} : 1 \leq i \leq \left\lfloor \frac{n}{3} \right\rfloor - 1 \right\} \cup \{u, v_1, v_2, v_{n-1}\}$$

with $|D| = \left\lfloor \frac{n}{3} \right\rfloor + 3$. Also for every vertex $v \in V(\widetilde{W_n}) - D$ is adjacent to at least two vertices of D, there exists $u \in D$ such that $|d_{\widetilde{W_n}}(v) - d_{\widetilde{W_n}}(w)| \leq 1$. Hence D is an equitable dominating set of $\widetilde{W_n}$. Further D is a total dominating set of $\widetilde{W_n}$ as $N(D) = V(\widetilde{W_n})$. Therefore D is a total equitable dominating set of $\widetilde{W_n}$.

For any $v \in D$, the set $D - \{v\}$ is not even an equitable dominating set of $\widetilde{W_n}$ as for all vertex $u \in V(\widetilde{W_n}) - (D - \{v\})$ there does not exist $w \in D - \{v\}$ such that $|d_{\widetilde{W_n}}(u) - d_{\widetilde{W_n}}(w)| \leq 1$. Therefore D is a minimal total equitable dominating set of $\widetilde{W_n}$.

Hence $\gamma_t^e(\widetilde{W_n}) = \left\lfloor \frac{n}{3} \right\rfloor + 3$, when $n \equiv 1(\bmod 3)$.

Conclusion

The concepts of total dominating set and total equitable dominating set are useful for the formation of any committee. It is desirable that each committee member might feel comfortable knowing at least one member of the committee. In this situation total domination is useful while there is no difference of opinion between any two members or they differ on at most one issue. In this situation the concept of equitable domination is applicable. We have investigated the total domination(total equitable domination) number of the graph obtained by switching of a vertex in standard graphs like P_n, C_n and W_n.

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References

[1] S. B. Anupama, Y. B. Maralabhavi and V. M. Goudar, *Cototal Edge Domination Number of a Graph*, Malaya Journal of Matematik, 4(2), (2016), 325 - 337.

[2] B. Basavanagoud, V. R. Kulli and V. V. Teli, *Equitable Total Domination in Graphs*, Journal of Computer and Mathematical Science, 5(2), (2014), 235 - 241.

[3] E. J. Cockayne, R. M. Dawes, S. T. Hedetniemi, *Total Domination in Graphs*, Networks, 10, (1980), 211 - 219.

[4] D. L. Grinstead and P.J. Slater, *Fractional domination and fractional packing in graphs*, Congressus Numerantium, 71, (1990), 153–172.

[5] T. W. Haynes, S. T. Hedetniemi and P. J. Slater , *Fundamentals of Domination in Graphs*, Marcel Dekker, New York, (1998).

[6] M. A. Henning, A. Yeo, *Total Domination in Graphs*, Springer, (2013).

[7] J. Joseline Manora and S. Veeramanikandan, *Intersection Graph on Non - Split Majority Dominating Graph*, Malaya Journal of Matematik, Special Issue(2), (2015), 476 - 480.

[8] A. R. Latheesh Kumar and V. Anil Kumar, *A Note on Global Bipartite Domination in Graphs*, Malaya Journal of Matematik, 4(3), (2016), 438 - 442.

[9] E.Sampathkumar, *The global domination number of a graph*, Journal of Mathematical and Physical Sciences, 23(5), (1989), 377-385.

[10] V. Swaminathan and K. M. Dharmalingam, *Degree Equitable Domination on Graphs*, Kragujevak Journal of Mathematics, 35(1), (2011), 191 - 197.

[11] S. K. Vaidya and A. D. Parmar, *Some New Results on Total Equitable Domination in Graphs*, Journal of Computational Mathematica, 1(1), (2017), 98 - 103.

[12] S. K. Vaidya and A. D. Parmar, *On Total Domination in Some Path Related Graphs*, International Journal of Mathematics and Soft Computing, 7(2), (2017), 103 - 109.

[13] S. K. Vaidya and A. D. Parmar, *Total Equitable Bondage Number of a Graph*, (Communicated).

[14] S. K. Vaidya and S. H. Karkar, *On Strong Domination Number of Corona Related Graphs*, Malaya Journal of Matematik, 5(4), (2017), 636 - 640.

[15] D. B. West, *Introduction to Graph Theory, 2/e*, Prentice - Hall of India, New Delhi, (2003).

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